

# PCO: Precision-Controllable Offset Surfaces with Sharp Features - Supplementary Material

LEI WANG, Shandong University, China  
 XUDONG WANG, Shandong University, China  
 PENGFEI WANG, Shandong University, China  
 SHUANGMIN CHEN\*, Qingdao University of Science and Technology, China  
 SHIQING XIN, Shandong University, China  
 JIONG GUO, Shandong University, China  
 WENPING WANG, Texas A&M University, USA  
 CHANGHE TU, Shandong University, China

## ACM Reference Format:

Lei Wang, Xudong Wang, Pengfei Wang, Shuangmin Chen, Shiqing Xin, Jiong Guo, Wenping Wang, and Changhe Tu. 2024. PCO: Precision-Controllable Offset Surfaces with Sharp Features - Supplementary Material. *ACM Trans. Graph.* 1, 1 (November 2024), 8 pages. <https://doi.org/10.1145/3687920>

## A PROOF OF THEOREM 4.1

For any pair of points  $\{p_1, p_2\}$  within the circumsphere of a tetrahedron  $T$ , their straight-line distance  $d(p_1, p_2)$  must satisfy

$$d(p_1, p_2) \leq 2R, \quad (1)$$

where  $R$  is the radius of the circumsphere. Therefore, if

$$\max_{k=1}^4 \{d^{(k)}(\mathcal{M}; T)\} > \delta + 2R \quad (2)$$

or

$$\min_{k=1}^4 \{d^{(k)}(\mathcal{M}; T)\} < \delta - 2R, \quad (3)$$

then  $T$  cannot contribute to the offset at  $\delta$ . Fig. 1 (a) and (b) illustrate two 2D examples for these scenarios respectively.

\*Corresponding author: Shuangmin Chen.

Authors' addresses: Lei Wang, Shandong University, Qingdao, Shandong, China, leiw1006@gmail.com; Xudong Wang, Shandong University, Qingdao, Shandong, China, sduwx0319@gmail.com; Pengfei Wang, Shandong University, Qingdao, Shandong, China, pengfei1998@foxmail.com; Shuangmin Chen, Qingdao University of Science and Technology, Qingdao, Shandong, China, csmqq@163.com; Shiqing Xin, Shandong University, Qingdao, Shandong, China, xinshiqing@sdu.edu.cn; Jiong Guo, Shandong University, Qingdao, Shandong, China, xhjia@amss.ac.cn; Wenping Wang, Texas A&M University, Texas, USA, wenping@tamu.edu; Changhe Tu, Shandong University, Qingdao, Shandong, China, chtu@sdu.edu.cn.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from [permissions@acm.org](mailto:permissions@acm.org).

© 2024 Association for Computing Machinery.  
 0730-0301/2024/11-ART \$15.00  
<https://doi.org/10.1145/3687920>

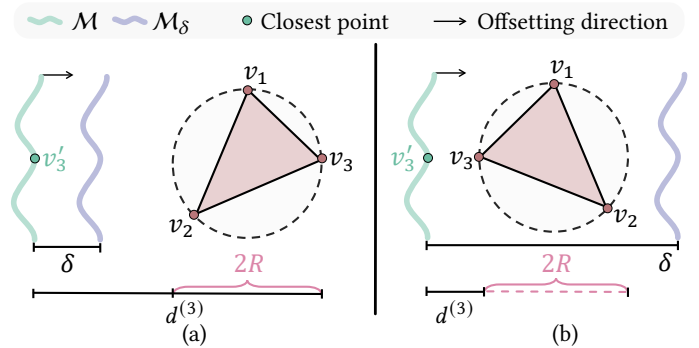


Fig. 1. In this 2D example, the triangle represents a 3D tetrahedron, while the base surface  $\mathcal{M}$  and the offset surface  $\mathcal{M}_\delta$  are depicted as green and purple curves, respectively. We abbreviate  $d^{(k)}(\mathcal{M}; T)$  as  $d^{(k)}$  for simplicity. In (a), the maximum vertex distance,  $d^{(3)}$ , exceeds  $\delta + 2R$ , indicating that the triangle does not contribute to the offset at  $\delta$ . In (b), the minimum vertex distance,  $d^{(3)}$ , is less than  $\delta - 2R$ , showing that the triangle also does not contribute to the offset at  $\delta$ .

## B ANALYSIS OF THE MERGED FIELD FORM

Considering the scenario where  $\delta > 0$  (with an analogous case for  $\delta < 0$ ). Let the merged field of  $\mathbf{D}_1$  and  $\mathbf{D}_2$  be denoted as  $\mathbf{D}_{12}$ . Define  $P$  and  $E$  as the sets of intersection points and edges of the tetrahedron  $T$  in  $\{T \cap \pi_1^+ \cap \pi_2^+\}$ , where  $\pi_u$  defined by  $\mathbf{D}_u = \delta$ .

If  $E = \emptyset$ , any  $p \in P$  must coincide with a vertex  $v_k$  of  $T$  ( $k = 1, 2, 3, 4$ ), implying  $\mathbf{D}_1(k) = \mathbf{D}_2(k) = \delta$ . Therefore,  $p \in \pi_{12}^-$ .

If  $E \neq \emptyset$ , let  $p \in P$  be the intersection of an edge  $e = (v_i, v_j) \in E$  with  $1 \leq i, j \leq 4$ . Then  $\pi_{12}$  must intersect with  $e$ , and we can deduce that  $f = f_i \cdot f_j = (\mathbf{D}_{12}(i) - \delta) \cdot (\mathbf{D}_{12}(j) - \delta) \leq 0$ .

(1) If  $f = 0$ :

- $f_i = 0$  and  $f_j = 0$ :  $\mathbf{D}_{12}(i) = \mathbf{D}_{12}(j) = \delta$ , any  $p$  on  $e$  satisfies  $p \in \pi_{12}^-$ .
- $f_i = 0$  and  $f_j \neq 0$ :  $p$  coincides with  $v_i$ , hence  $p \in \pi_{12}^-$ .
- $f_j = 0$  and  $f_i \neq 0$ :  $p$  coincides with  $v_j$ , hence  $p \in \pi_{12}^-$ .

(2) If  $f < 0$ :

Consider the function  $h(x, y; \delta)$ :

$$\begin{cases} \frac{\delta-x}{y-x} & (-\infty, \delta] \times [\delta, +\infty) \setminus (\delta, \delta) \\ 0 & (x, y) = (\delta, \delta) \end{cases} \quad (4)$$

The partial derivatives of  $h$  with respect to  $x$  and  $y$  are  $h_x < 0$  and  $h_y < 0$  respectively for  $x \in (-\infty, \delta), y \in (\delta, +\infty)$ . Suppose  $p^*$  is the intersection of  $\pi_{12}$  on  $e$ , its position can be calculated as

$$p^* = v_i + h(\mathbf{D}_{12}(j), \mathbf{D}_{12}(i); \delta)(v_j - v_i). \quad (5)$$

Given that  $\mathbf{D}_{12} = \mathbf{S} \odot \min(|\mathbf{D}_1|, |\mathbf{D}_2|)$ , if  $\mathbf{S} > 0$ , point  $p$  must lie below the intersection  $p^*$ .

In conclusion, the half-plane  $\pi_{12}$  defined by  $\mathbf{D}_{12} = \delta$  satisfies

$$\begin{aligned} \{T \cap \pi_{12}^+\} &\subset \{T_i \cap \pi_1^{++} \cap \pi_2^{++}\} && \text{if } \delta > 0 \text{ and } \mathbf{S} > 0, \\ \{T \cap \pi_{12}^-\} &\subset \{T_i \cap \pi_1^{--} \cap \pi_2^{--}\} && \text{if } \delta < 0 \text{ and } \mathbf{S} < 0. \end{aligned} \quad (6)$$

## C IMPLEMENTATION DETAILS

*Distance Calculator.* Our algorithm supports both signed and unsigned distance computations.

For the direct computation of signed distances, the main challenge lies in determining the sign of a point  $p$  (e.g., a vertex of a tetrahedron). In our implementation, we employed several methods for this purpose, including pseudo-normals [Bærentzen and Aanaes 2005], ray intersection [Cherchi et al. 2022], and winding number [Barill et al. 2018; Jacobson et al. 2013]. Each method presents a trade-off between robustness and efficiency. However, these approaches struggle with imperfect meshes, such as non-manifold, or non-watertight meshes, due to undefined orientations. In such cases, we recommend using unsigned distance computations, which yield two offset results meanwhile—inward and outward. Since our results retain excellent properties of being watertight, manifold, and free of self-intersections, we can accurately extract the internal and external results using straightforward sign determination methods.

*Analysis of Half-plane Cutting Process.* Our incremental half-plane cutting process can become time-consuming if all contributing triangles are included in the distance field computation within a tetrahedron. To mitigate this, we implement an effective triangle filtering process.

Before delving into details, we first define the "competitive relationship" between two linear distance fields  $\mathbf{D}_1$  and  $\mathbf{D}_2$  within a tetrahedron  $T$ .

*Definition C.1.* The distance field  $\mathbf{D}_1$  is defeated by  $\mathbf{D}_2$  only when  $\{\pi_1^+ \cap T\} \cap \{\pi_2^+ \cap T\} = \{\pi_2^+ \cap T\}$ , where  $\pi_i^+$  is the positive side of half-plane  $\pi_i$  defined by  $\pi_i = \delta, i = 1, 2$ .

If one distance field is defeated by another, it is not considered valid for participation in the half-plane cutting process. Based on this definition, we propose the following theorem:

**THEOREM C.2.** *If the distance field  $\mathbf{D}_1$  is defeated by  $\mathbf{D}_2$ , it must satisfies  $\min |\mathbf{D}_1| > \max |\mathbf{D}_2|$ .*

Further, let  $C_i^T$  denote the contributing triangles of  $T$ :

$$C_i^T = \{t | t \text{ has the contribution to } T\}.$$

According to the above theorem C.2, we can deduce the following lemma:

**LEMMA C.3.** *Let  $\text{Prj}_{t_i}$  be the projected triangle on the original mesh of vertex  $v_i (i = 1, 2, 3, 4)$  in  $T$ , where  $\text{Prj}_{t_i} \in C_i^T$ . The triangle is impossible defeated by other triangles in  $C_i^T$ .*

The proof is straightforward: the field  $|\mathbf{D}_{\text{Prj}_{t_i}}|$  at least has one minimum value compared with other distance fields whose source triangles are belong to  $C_i^T$ .

Following the above theories, we first compute the distance fields  $\mathbf{D}_{\text{Prj}_{t_i}}$  from the triangles  $\text{Prj}_{t_i}$ . Subsequently, we maintain the maximum distance values of four vertexes in  $T$  using a quadruple  $\mathbf{M}_T$ , initialized as

$$\mathbf{M}_T(j) = \max_{i=1,2,3,4} \{\mathbf{D}_{\text{Prj}_{t_i}}(j)\}, j = 1, 2, 3, 4. \quad (7)$$

Only the newly distance field  $|\mathbf{D}|$  satisfies

$$\max |\mathbf{D}| < \min |\mathbf{M}_T|, \quad (8)$$

it can be considered as a valid one to involve in the subsequent half-plane cutting process.

## D EXPERIMENTAL DETAILS AND RESULTS

### D.1 Evaluation metrics

We use relative one-sided Chamfer distance ( $d_C$ ), relative one-sided Hausdorff distance ( $d_H$ ), Mean Absolute Normal deviation ( $N_{\text{MAE}}$ ), and Normal Score (N-Score) to evaluate the accuracy of offsetting results. We denote  $r = |\delta|$ ,  $\mathcal{M}_\delta$  and  $\mathcal{M}$  as the offsetting mesh at  $\delta$  and the input mesh, respectively. Let  $P_1$  and  $P_2$  be the randomly sampled points from  $\mathcal{M}_\delta$  and  $\mathcal{M}$ .

*Distance error metrics.* The relative one-sided ( $P_1 \rightarrow P_2$ ) Chamfer distance and Hausdorff distance between two point clouds  $P_1$  and  $P_2$  are defined as follows:

$$d_C = \frac{1}{r * |P_1|} \sum_{p_1 \in P_1} \left| \min_{p_2 \in P_2} d(p_1, p_2) - \delta \right| \quad (9)$$

$$d_H = \frac{1}{r} \max_{p_1 \in P_1} \min_{p_2 \in P_2} |d(p_1, p_2) - \delta|,$$

where  $d(p_1, p_2)$  is the straight-line distance between points  $p_1, p_2$ . We use the  $L - 1$  norm following [Wang and Manocha 2013; Zint et al. 2023].

*Normal consistency metrics.* The metrics of Mean Absolute Error of normal deviation and Normal Score at a given threshold  $\gamma$  between two point clouds  $P_1$  and  $P_2$  are defined as follows:

$$\begin{aligned} N_{\text{MAE}} &= \frac{1}{|P_1|} \sum_{p_1 \in P_1} \text{angle}(\mathbf{n}_{p_1}, \mathbf{n}_{\text{closest}(p_1, P_2)}) \\ \text{N-Score} &= \frac{|\{\text{angle}(\mathbf{n}_p, \mathbf{n}_{\text{closest}(p_1, P_2)}) < \gamma\}|}{|P_1|}, \end{aligned} \quad (10)$$

where

$$\text{closest}(p, P) = \arg \min_{p' \in P} (p, p'), \quad (11)$$

and

$$\text{angel}(\mathbf{n}_p, \mathbf{n}_{p'}) = \arccos(\mathbf{n}_p \cdot \mathbf{n}_{p'}). \quad (12)$$

In our experiments,  $\gamma = 5^\circ$ .

### D.2 Validation on Triangle Soup

Our algorithm exhibits impressive stability with respect to noisy triangle mesh - triangle soups, consistently ensuring valid output, as shown in Fig. 2.

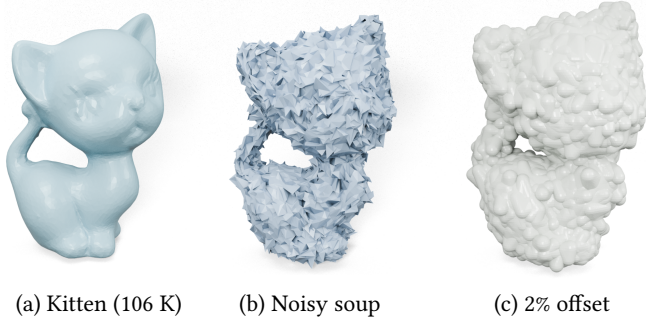


Fig. 2. Our method effectively processes triangle soups and ensures that the output is topology correct.

### D.3 Integrated with Alpha Shape

When the complexity of and the input the offset distance are all very large, our algorithm may become time-consuming due to the large number of contributing triangles involved in distance field computation in each tetrahedron. However, leveraging our algorithm’s effective sharp feature preservation, we offer an alternative approach to achieve efficiency without significantly sacrificing accuracy: initially employing alpha shape algorithm [Edelsbrunner and Mücke 1994] to compute a coarse result at a smaller offset distance, followed by applying our method.

As illustrated in Fig. 3, this decreases processing time by nearly 90% while still effectively maintaining the clarity of the feature lines.

Table 1. Quantitative comparison results on twisted and thin-plate models. The **best** scores are highlighted in bold with underlining, while the **second best** scores are highlighted in bold.

		94884		104559		213887	
		-0.5	1	-0.5	2	-0.5	0.5
Time/s	AW	-	365.96	-	204.38	-	203.45
	DC	257.84	268.63	142.77	146.02	207.07	211.78
	FPO	-	151.99	-	372.81	-	151.99
	HSP	<b>23.64</b>	<b>64.85</b>	<b>7.30</b>	<b>40.84</b>	<b>9.37</b>	<b>15.25</b>
	<b>Ours</b>	<b>5.08</b>	<b>8.64</b>	<b>1.86</b>	<b>11.67</b>	<b>2.22</b>	<b>4.25</b>
$d_C (\times 10^{-3}) \downarrow$	AW	-	<b>0.780</b>	-	<b>0.553</b>	-	<b>2.152</b>
	DC	<b>1.214</b>	1.645	<b>5.845</b>	2.045	<b>1.188</b>	<b>4.180</b>
	FPO	-	9.040	-	8.448	-	19.851
	HSP	<b>10.833</b>	5.441	28.595	17.734	18.501	24.247
	<b>Ours</b>	20.299	<b>0.138</b>	<b>2.593</b>	<b>0.523</b>	<b>13.508</b>	31.296
$d_H (\times 10^{-3}) \downarrow$	AW	-	<b>38.080</b>	-	38.756	-	<b>153.300</b>
	DC	<b>61.040</b>	46.821	<b>127.980</b>	<b>30.758</b>	<b>45.040</b>	<b>133.030</b>
	FPO	-	353.290	-	85.453	-	418.840
	HSP	399.820	170.280	795.140	1953.400	733.940	163.360
	<b>Ours</b>	<b>149.560</b>	<b>38.112</b>	<b>8.293</b>	<b>38.200</b>	<b>162.700</b>	206.760
$N_{MAE} \downarrow$	AW	-	<b>12.341</b>	-	28.424	-	16.982
	DC	<b>2.958</b>	12.565	<b>9.561</b>	30.731	<b>2.287</b>	15.892
	FPO	-	32.765	-	30.451	-	35.235
	HSP	8.464	12.499	24.663	<b>30.620</b>	7.712	<b>15.343</b>
	<b>Ours</b>	<b>4.978</b>	<b>2.358</b>	<b>7.172</b>	<b>1.732</b>	<b>3.314</b>	<b>4.159</b>
N-Score/% $\uparrow$	AW	-	74.00	-	32.30	-	65.39
	DC	<b>93.28</b>	<b>74.80</b>	<b>73.13</b>	<b>34.30</b>	<b>92.34</b>	<b>68.12</b>
	FPO	-	54.91	-	30.40	-	59.76
	HSP	<b>81.77</b>	73.00	2.80	28.70	62.55	64.20
	<b>Ours</b>	64.67	<b>82.15</b>	<b>44.82</b>	<b>97.05</b>	<b>76.61</b>	<b>71.48</b>

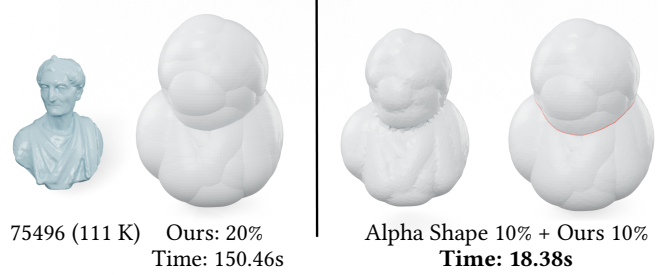


Fig. 3. To effectively manage time and ensure accurate results when dealing with complex models and significant offset distances, our method can be integrated with Alpha Shape [Edelsbrunner and Mücke 1994]. In this scenario, features are still accurately recovered with minimal time expenditure.

### D.4 Comparison on CAD Models

We give the comparison statistics in Table 2. The corresponding visual comparison is given in Fig. 4, Fig. 5, and Fig. 6. It can be seen that our method achieves effectively balances accuracy and robustness, outperforming alternative methods in preserving fine features while maintaining overall structural integrity.

### D.5 Comparison on Twisted and Thin-plate Models

The quantitative comparison statistics are reported in Table 1, while the visual comparison is available in Fig. 7. The comparison shows that our method is better at recovering thin geometry features and can achieve a good trade-off between smoothness and feature preservation.

### D.6 Comparison on Models Reconstructed from Large Raw Scan Data

We show the visual comparison of different approaches with varying offset distance in Fig. 8, and the qualitative results are shown in Table 3. Both qualitative and quantitative comparisons show that our method can faithfully recover fine geometric details and thin structures, outperforming the other methods.

## REFERENCES

- Jakob Andreas Bærentzen and Henrik Aanaes. 2005. Signed distance computation using the angle weighted pseudonormal. *IEEE Transactions on Visualization and Computer Graphics* 11, 3 (2005), 243–253.
- Gavin Barill, Neil G Dickson, Ryan Schmidt, David IW Levin, and Alec Jacobson. 2018. Fast winding numbers for soups and clouds. *ACM Transactions on Graphics (TOG)* 37, 4 (2018), 1–12.
- Gianmarco Cherchi, Fabio Pellacini, Marco Attene, and Marco Livesu. 2022. Interactive and Robust Mesh Booleans. *ACM Trans. Graph.* 41, 6, Article 248 (nov 2022), 14 pages. <https://doi.org/10.1145/3550454.3555460>
- Herbert Edelsbrunner and Ernst P Mücke. 1994. Three-dimensional alpha shapes. *ACM Transactions On Graphics (TOG)* 13, 1 (1994), 43–72.
- Alec Jacobson, Ladislav Kavan, and Olga Sorkine-Hornung. 2013. Robust inside-outside segmentation using generalized winding numbers. *ACM Transactions on Graphics (TOG)* 32, 4 (2013), 1–12.
- Charlie CL Wang and Dinesh Manocha. 2013. GPU-based offset surface computation using point samples. *Computer-Aided Design* 45, 2 (2013), 321–330.
- Daniel Zint, Nissim Maruani, Mael Rouxel-Labbé, and Pierre Alliez. 2023. Feature-Preserving Offset Mesh Generation from Topology-Adapted Octrees. In *Computer Graphics Forum*. 12.

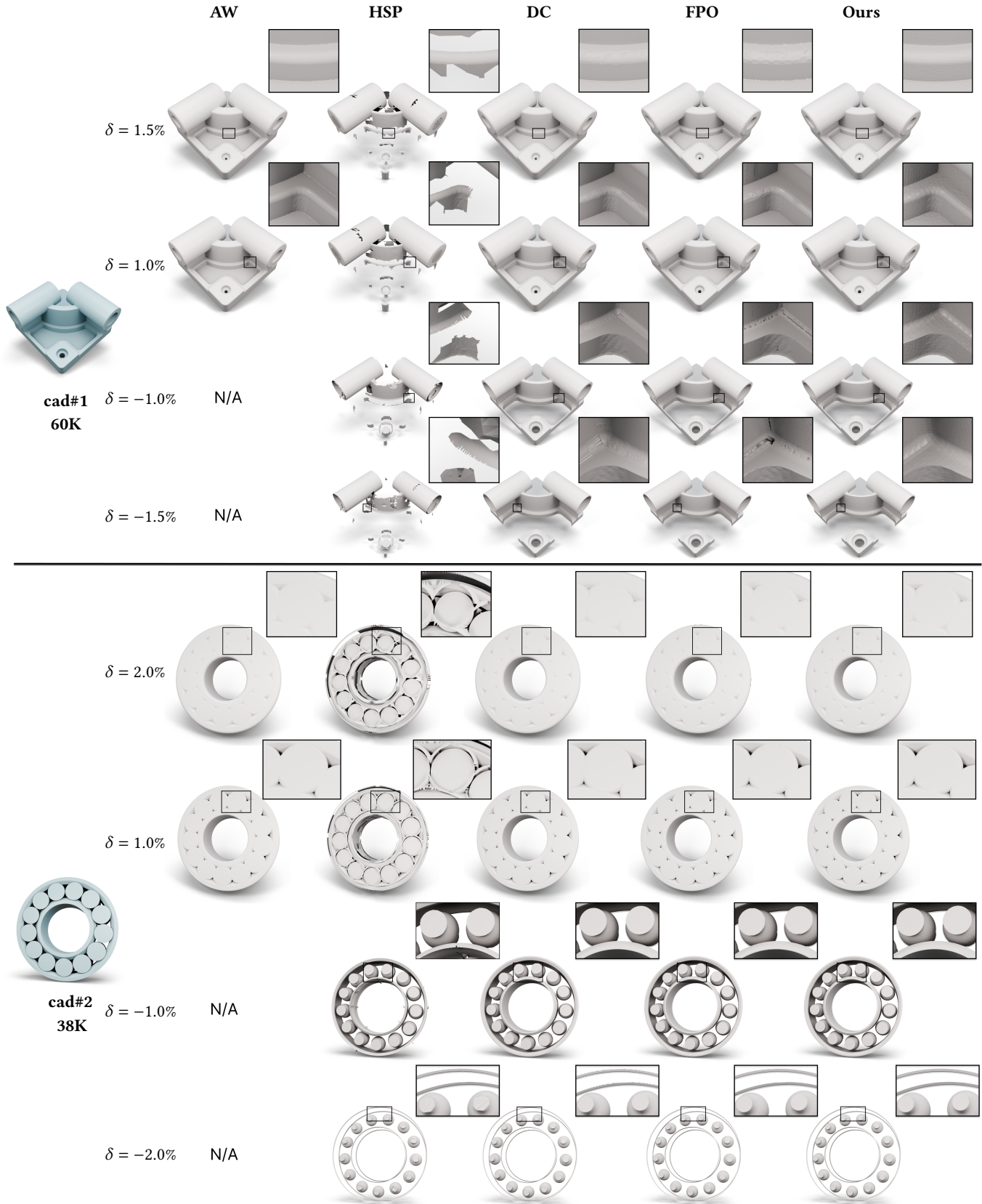


Fig. 4. Visual comparison results on cad models (part one).

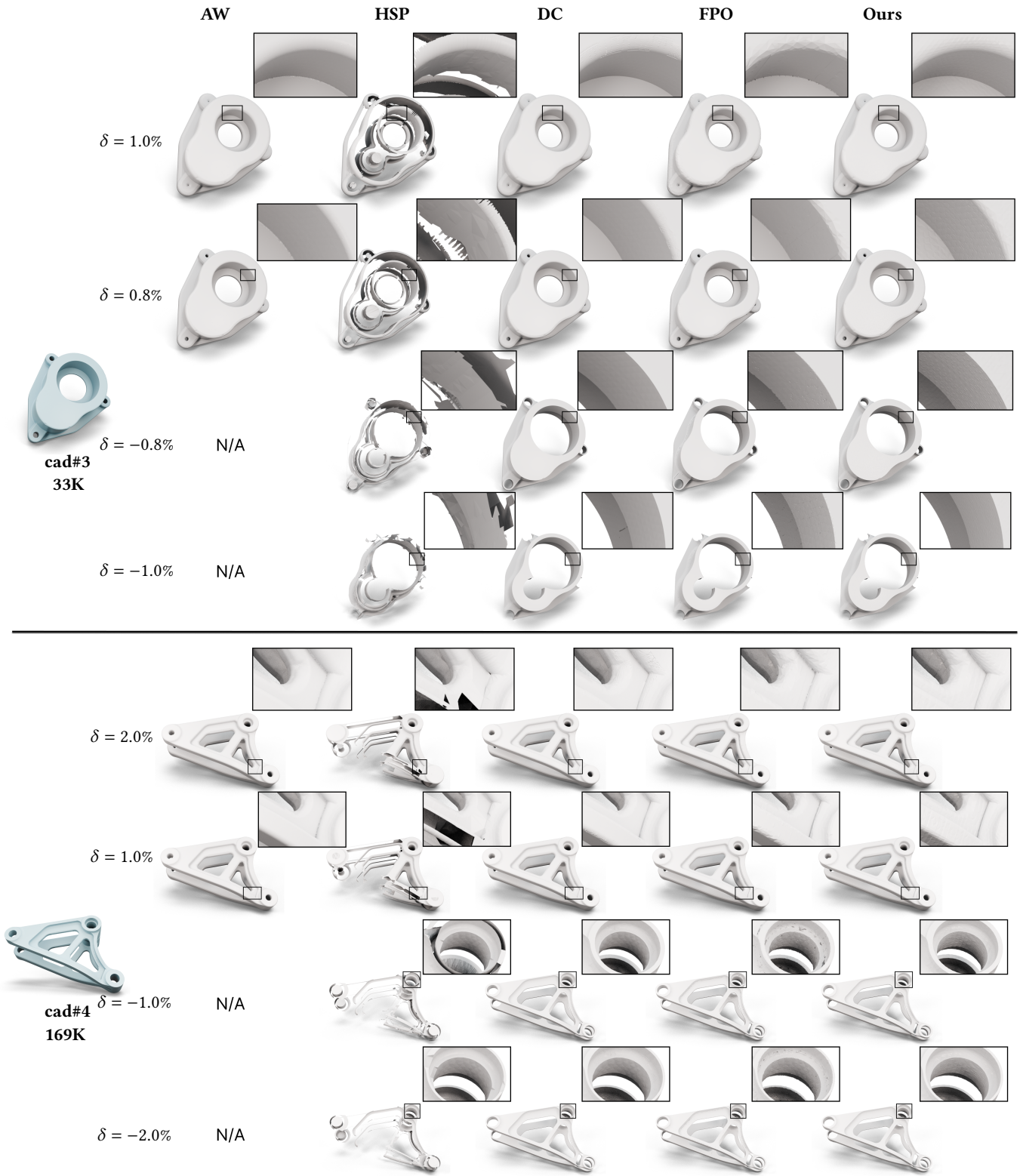


Fig. 5. Visual comparison results on cad models (part two).

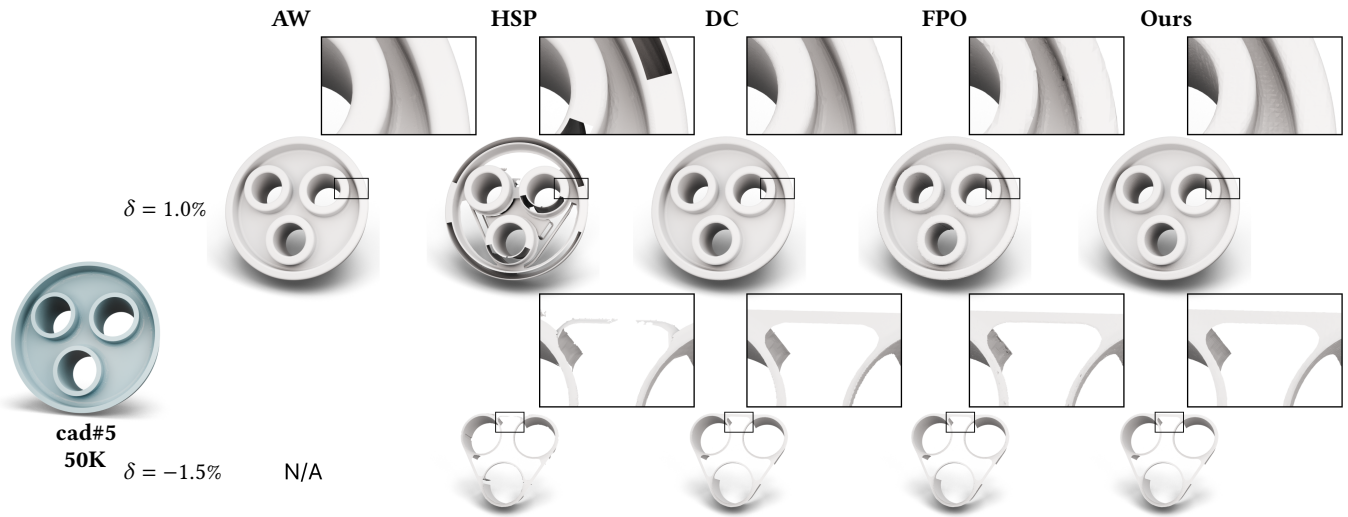


Fig. 6. Visual comparison results on cad models (part three).

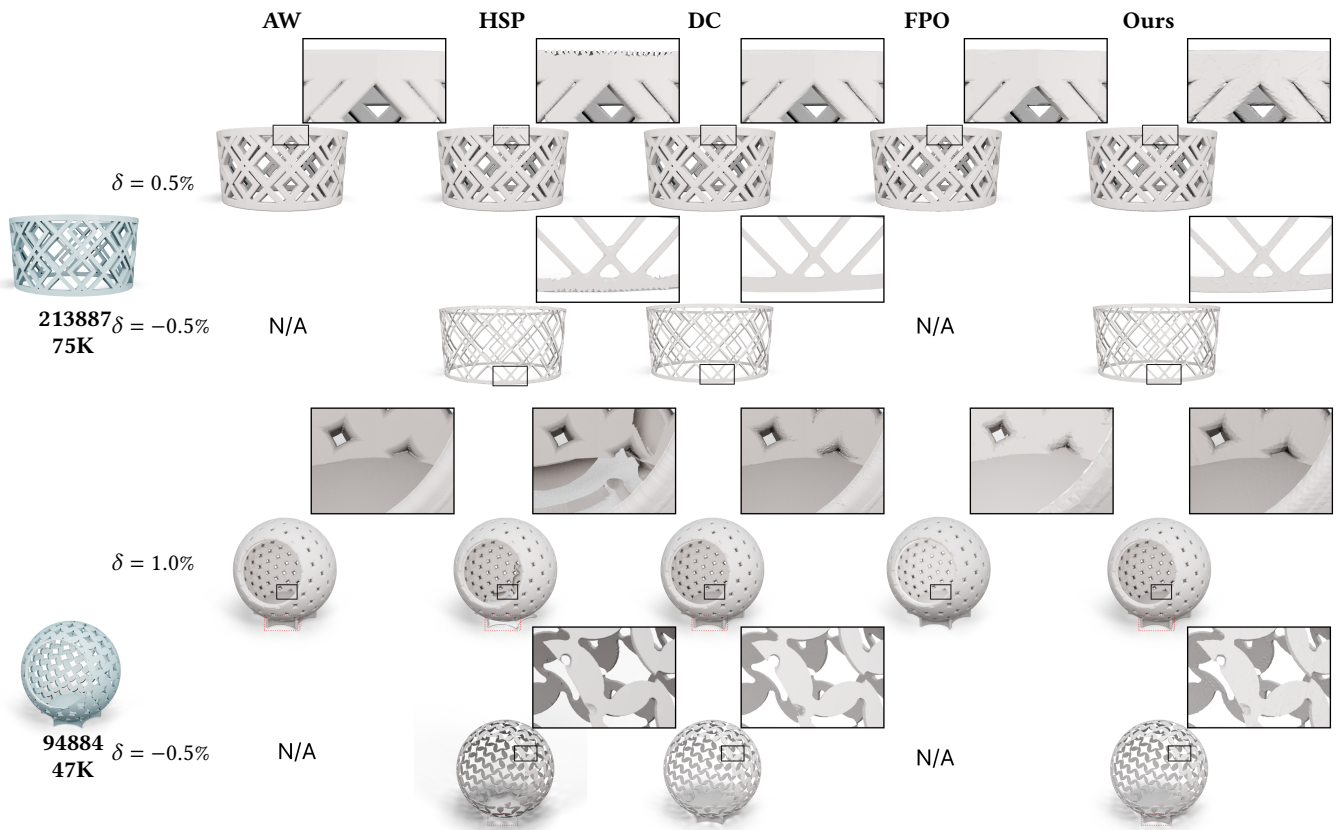


Fig. 7. Visual comparison results on twisted and thin-plate models.

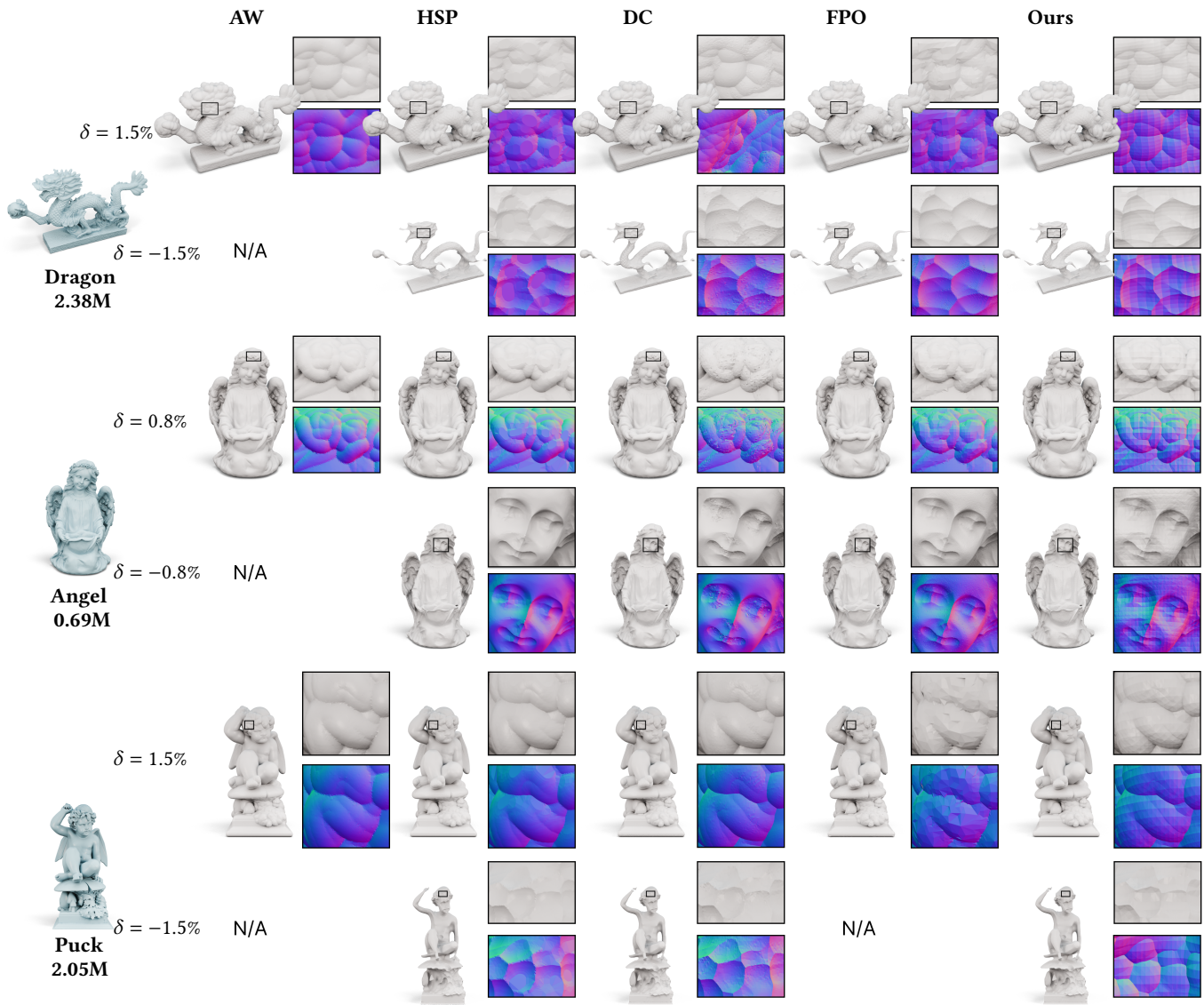


Fig. 8. Visual comparison results on models reconstructed from large raw scans.

