Supplementary Material: Field-Aligned Online Surface Reconstruction

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This supplementary document contains additional information for the main paper [Schertler et al. 2017]. Specifically, this document describes the calibration process of the scanner and the Vive tracking system as well as the definition of the field-aligned smoothing brush.

1 CALIBRATION

Vive-To-Scan. The Vive tracking system provides an affine model transformation matrix $M_{V \to C_1}$ for the tracked controller. This transform can be used to place the acquired 3D scan (defined in the rig's coordinate system) in the Vive's coordinate system, which is done by applying the model transform $M_{V \to S}$ to the scan:

$$M_{V \to S} = M_{V \to C_1} \cdot M_{C_1 \to S} \tag{1}$$

The matrix $M_{C_1 \to S}$ is the transform from the controller's to the scan's coordinate system and is the result of the calibration process.

In order to calibrate this matrix, we take a scan of a second controller, whose position and orientation we also track. We then let the user define three correspondences between this scan and the controller's CAD model. These correspondences allow us to calculate a coarse alignment matrix $M_{C_2 \to S}$, which we refine using Sparse ICP. The calibration matrix is then:

$$M_{C_1 \to S} = M_{V \to C_1}^{-1} \cdot M_{V \to C_2} \cdot M_{C_2 \to S} \tag{2}$$

Turntable. To place scans in the Vive's coordinate system and account for the rotation of the turntable, the associated rotation axis (represented as the axis direction and a position on the axis $(d_a \in \mathbb{R}^3, p_a \in \mathbb{R}^3)$) needs to be calibrated. Once these parameters are known, we can modify the model transform to place a scan as:

$$M_{V \to S} = T(p_a) \cdot R_{d_a}(\alpha)^{-1} \cdot T(p_a)^{-1} \cdot M_{V \to C_1} \cdot M_{C_1 \to S}, \tag{3}$$

where T is a translation matrix and $R_{d_a}(\alpha)$ is the rotation matrix about the given axis and angle α. Angle α is the current rotation of the turntable, which we can define through the turntable's API.

In order to calibrate the rotation axis, we place a controller on the turntable and let it rotate, recording the controller's model transform M_i after every 45°. For every pair of opposite recordings (M_i, M_{i+4}), we find the associated axis of the rotation matrix $R_{i+4} \cdot R_i^{-1}$ (R_i represents the linear part of M_i) and average all of them to define d_a.

Once the axis direction is found, we calculate p_a by solving the following linear least squares problem:

$$\begin{aligned} d_i &:= \frac{O_i - O_{i+4}}{\|O_i - O_{i+4}\|} \\ c_i &:= \frac{1}{2}(O_i + O_{i+4}) \\ p_a &= \arg\min_p \sum_i \langle p - c_i, d_i \rangle^2 \\ &= \arg\min_p \sum_i (d_i^T \cdot p - \langle c_i, d \rangle)^2, \end{aligned} \tag{4}$$

where $O_i = M_i \cdot v$ specifies the global position of some local position v on the controller (we use the controller's tip and ensure that the tip does not lie on the axis during calibration). This system solves for the point p_a that lies on the intersection of the bisectors of the lines connecting every pair's controller tips. Although these connecting lines should be the associated circle's diameter, we treat them only as secants because the turntable does not allow rotation of exactly 180° due to the step motor's resolution. We solve Equation 4 in the 2D subspace that is orthogonal to the rotation axis.

2 FIELD-ALIGNED BRUSHES.

The direction-aligned smoothing brush applies a Laplacian smoothing to the selected region, where the derivative vector is scaled with a user-defined strength along the principal axes of the local tangent plane. More specifically, given a point p with its normal n_p and the chosen direction o_p, we calculate the new position as:

$$\begin{aligned} d &\leftarrow \sum_{n \in N(p)} \omega(p - n) \cdot n - p \\ p &\leftarrow p + T^T \cdot S \cdot T \cdot d, \end{aligned} \tag{5}$$

where $N(p)$ are the positions of the neighbors of point p with Gaussian weights $\omega(\cdot)$ that sum to 1. The transformation matrix $T \in \mathbb{R}^{3\times3}$ transforms the result of the Laplacian d into the local frame of point p (i.e. its column vectors are the chosen direction, the orthogonal direction, and the normal), and $S \in \mathbb{R}^{3\times3}$ is a diagonal scaling matrix, where the entries on the diagonal correspond to user-defined smoothing strengths in the three principal directions.

REFERENCES

Nico Schertler, Marco Tarini, Wenzel Jakob, Misha Kazhdan, Stefan Gumhold, and Daniele Panozzo. 2017. Field-Aligned Online Surface Reconstructions. *ACM Transactions on Graphics* 36 (2017).