# Consistent normals for large point clouds 

 Global optimality for streaming orientation of point cloud normals
## | Introduction



Consistent normal orientations are essential for renderings and many processing algorithms.
Given a point cloud with unoriented normals, we re-orient the normals in a consistent way.
Key points of our work:

- Consolidation of previous work into a Markov Random Field model
- Globally optimal solution of the MRF
- Out-of-core framework for large point clouds


## | Problem formalization

Every point $p_{i}$ with normal $n_{i}$ is assigned a label $l_{i} \in\{-1,1\}$, such that the adapted normals $n_{i}^{\prime}:=l_{i} n_{i}$ are most consistent.
Consistency is measured upon a neighbor graph with edges $\mathcal{E}$ and the flip criterion $\phi: \varepsilon \rightarrow \mathbb{R}$ (positive for consistent edges, negative for non-consistent edges). Hoppe's flip criterion [1] can be expressed as:

$$
\phi_{\text {Hoppe }}(i, j)=\left\langle n_{i}, n_{j}\right\rangle
$$

Example graph:


The following potentials are derived from this flip criterion: $E_{i, j}\left(l_{i}, l_{j}\right)=$
$\left\{\begin{array}{ll}|\phi(i, j)| \cdot \omega\left(p_{i}, p_{j}\right) & \begin{array}{l}\phi(i, j) \geq 0 \\ 0\end{array} \\ \text { otherwise }\end{array} \oplus l_{i}=l_{j}\right.$ $\omega\left(p_{i}, p_{j}\right)=1-\frac{\operatorname{distance}\left(p_{i}, p_{j}\right)^{2}}{r^{2}}$

This potential definition can be visualized as follows:


The sum of potentials forms a labeling's energy. The optimal
orientation is the energy's minimizer: orientation is the energy's minimizer:

$$
\begin{align*}
E(L) & =\sum_{\{i, j\} \in \mathcal{E}} E_{i, j}\left(L_{i}, L_{j}\right)  \tag{1}\\
L^{*} & =\underset{L}{\arg \min } E(L)
\end{align*}
$$

The nearby sections give an overview of various ways to solve this equation.

## Signed Union Find

An approximate solution of (1) can be found by propagating the orientation along the minimum spanning tree [1].
The MST can be calculated with Kruskal's algorithm and the Union Find data structure [2].
We augment this structure with a sign bit, which enables fast flips of entire connected components. Thus, MST propagation can be executed on-the-fly without explicit MST calculation.

$\qquad$

Components can be flipped by inverting their root's sign bit. Merging components requires the sign bit update $s_{i} \leftarrow \mathrm{~s}_{\mathrm{i}} \oplus \mathrm{s}_{\mathrm{i}}$.


Similar updates exist for path compression. Sign bits are unaffected by union-by-rank.

## Results

Using a global optimizer yields more consistent results than the greedy MST approach:
Hoppe's flip criterion:


Our out-of-core approach allows orientation of large data sets and higher performance even for medium-sized data sets.

## | Globally optimal solution with QPBO

Greedy spanning tree-based solutions fail in cases with con tra dicting edges like in this simple example:


## Distance-weighted output of flip criterion

MST solution, $\mathrm{E}=2.4 \quad$ Optimal solution, $\mathrm{E}=2.0$
Instead, we use QPBO (quadratic pseudo-boolean optimization [4]) to find a globally optimal solution.
OPBO outputs a labeling with $l_{i} \in\{-1,1, \varnothing\}$, where $\varnothing$ denotes an unlabeled node. Every labeled node is part of a globally optimal solution (partial optimality property).
In order to remove the solution ambiguity, we force the first element of every connected component to the label +1 by altering the energy as follows:

$$
E_{Q P B O}:=E+\sum_{c \in C} \begin{cases}0 & l_{c_{0}}=+1 \\ \zeta & \text { otherwise }\end{cases}
$$

for the set of connected components C and an arbitrary positive number $\zeta$.
We use the variant QPBO-I (OPBO Improve) to find labels for formerly unlabeled nodes. In this process, we successively fix random nodes to their according MST solution and re-solve OPBO until all nodes are labeled. We refer to this method as MST+QPBO-I.

## Orientation of large point clouds

Big data sets are first sorted and Data in memory [5]. This allows out-of-core processing with a small slice of data in memory.


In a first step, the point cloud is segmented into locally orientable mented into locally orientable tradicting edges within a single patch.
The global patch orientation is then calculated on the reduced neighbor graph, where points of the same patch have been contracted to a single node. The energy functions of edges are summed during this contraction.

Segmentation Details
When a point is processed, all of its left neighbors are already segmented. The segments of the closest neighbors are considered as the segment for the processed node.
The sum of distance-weighted flip criterion output for each segment is the segment's vote. The segment with the greatest absolute vote wins:


Vote for purple segment

Vote for yellow segment

For the reduction to be energy-preserving, each considered segment must fulfill the intra-segment criterion and inter-segment criterion.

The intra-segment criterion constrains edges from neigh bors within the considered segment. Every edge must have the same sign. We allow a small tolerance to reduce the number of segments.
The inter-segment criterion constrains edges from neighbors in other segments. They must have the same sign as must have the same sign as the two segments. We allow a small tolerance to reduce the number of segments.
If no eligible segment exists, a new segment is created

## References

[1] Hoppe H. et al.: Surface reconstruction from unorganized points. In Proceedings of the 19th Annual Conference on Computer Graphics and Interactive Techniques (New York, NY, USA, 1992), SIGGRAPH '92, ACM, pp. 71-78.
[2] Kruskal J. B.: On the shortest spanning subtree of a graph and the traveling salesman problem. Proceedings of the American Mathematical society 7, 1 (1956), 48-50
[3] Xie H. et al.: Piecewise c1 continuous surface reconstruction of noisy point clouds via local implicit quadric regression. In Visualization, 2003. VIS 2003. IEEE, pp. 91-98
[4] Rother C. et al.: Optimizing binary MRFs via extended roof duality. In Computer Vision and Pattern Recognition, 2007. CVPR'07. IEEE Conference on (2007), IEEE, pp. 1-8.
[5] Pajarola R.: Stream-processing points. In Visualization, 2005. VIS 05. IEEE (Oct 2005), pp. 239-246.

## Acknowledgements

The Dragon data have been provided by the Stanford 3D Scanning Repository. The Pegasus model has been provided by the AIM @SHAPE-VISIONAIR Shape Repository. Data provision is thankfully acknowledged.
This work is partially funded by the European Social Fund and the Free State of Saxony (ESF project numbe 100226943, "ADFEX")


