## Definitions and Cayley Tables

- What is a (sub)group, a group law and the group axioms? Name some examples.
- What is a group multiplication table (a. k. a. Cayley table)? As an example, fill such a table (if possible) for the following groups and name their identity and inverse elements:
  1. $(G_1, \cdot) = (\{1, -1, i, -i\}, \cdot)$,
  2. $(G_2, \cdot) = (\mathbb{R}^x, \cdot)$ with $\mathbb{R}^x \equiv \mathbb{R} \setminus \{0\}$,
  3. $(G_3, \cdot) = (\mathbb{R}, +)$,
  4. $(G_4, \cdot) = (\mathbb{Z}/m\mathbb{Z}, +)$, where $\mathbb{Z}/m\mathbb{Z}$ is the set of all sections modulo $m$, i. e. all sections of the form $[a]_m = a + m\mathbb{Z} = \{b | b \equiv a \bmod (m)\}$, ($\hat{=}$ $m$-fold rotations). Show for $m = 4$.
  5. $(G_5, \cdot)$ = point group $C_n^k$, i. e. rotations about an $n$-fold symmetry axis $\hat{=}$ rotation by angles $\varphi = k\frac{2\pi}{n}$. Show for $n = 3$ .

  Which of them are abelian groups? What subgroups can you identify?
  If the modulo operation and residue classes are new to you, have a look here:
  https://de.wikipedia.org/wiki/Restklasse
  https://groupprops.subwiki.org/wiki/Group_of_integers_modulo_n
- Have a look at the Cayley table for the point group $C_{3v}$. What can you tell about its properties, i. e. corresponding symmetry elements, number of symmetry transformations, commutativity (abelian group?), possible duplicates?

| $C_{3v}$ | $E$ | $C_3$ | $C_3^2$ | $\sigma_{v1}$ | $\sigma_{v2}$ | $\sigma_{v3}$ |
|---|---|---|---|---|---|---|
| $E$ | $E$ | $C_3$ | $C_3^2$ | $\sigma_{v1}$ | $\sigma_{v2}$ | $\sigma_{v3}$ |
| $C_3$ | $C_3$ | $C_3^2$ | $E$ | $\sigma_{v2}$ | $\sigma_{v3}$ | $\sigma_{v1}$ |
| $C_3^2$ | $C_3^2$ | $E$ | $C_3$ | $\sigma_{v3}$ | $\sigma_{v1}$ | $\sigma_{v2}$ |
| $\sigma_{v1}$ | $\sigma_{v1}$ | $\sigma_{v3}$ | $\sigma_{v2}$ | $E$ | $C_3^2$ | $C_3$ |
| $\sigma_{v2}$ | $\sigma_{v2}$ | $\sigma_{v1}$ | $\sigma_{v3}$ | $C_3$ | $E$ | $C_3^2$ |
| $\sigma_{v3}$ | $\sigma_{v3}$ | $\sigma_{v2}$ | $\sigma_{v1}$ | $C_3^2$ | $C_3$ | $E$ |

- Fill the Cayley table for the point group $C_{2v}$ with help of the $H_2O$ or 1,3-Dichlorobenzene molecule as an example.

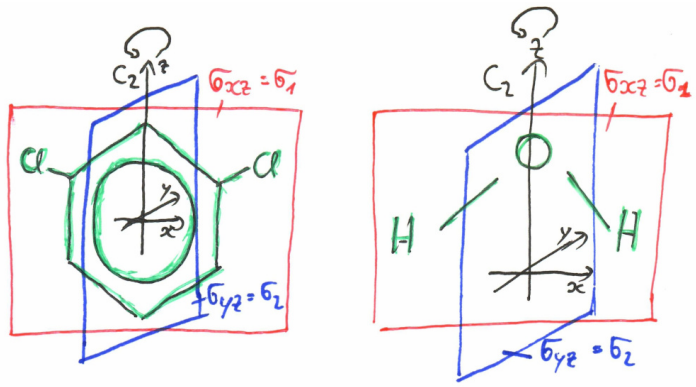


Figure 1: $H_2O$ and 1,3-Dichlorobenzene are molecules illustrating symmetries according to the point group $C_{2v}$.

- What is the symmetric group? Construct the Cayley table for the symmetric group $S_3$. What is this group's order and degree? How is it connected to the Cayley theorem?