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APPLICATION OF TEMPERATURE SOLUTIONS FOR FORCED CONVECTION SYSTEMS WITH VOLUME HEAT SOURCES TO GENERAL CONVECTION PROBLEMS

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by

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SUMMARY

This report concerns itself with the application of previously developed mathematical temperature solutions for forced convection systems having volume heat sources within the fluids to more general convection problems Convection solutions are tabulated so that it is possible to determine the detailed radial temperature structure within fluids having uniform volume heat sources and being uniformly cooled at the duct walls, the detailed temperature profile of a specific system is presented The derivation of equations describing the temperature structure and heat transfer rates in a duct system in which the wall is nonuniformly cooled is given, the temperature structure of a specific heat exchange system is also presented

NOMENCLATURE

Letters

A	cross sectional heat transfer area, ft^2
c_p	fluid heat capacity, Btu/lb ^oF
c_{pc}	heat capacity of coolant, Btu/lb ^oF
c_{pf}	heat capacity of volume-heat source fluid, Btu/lb ^oF
h	heat transfer conductance or coefficient, Btu/hr ft^2 ^oF
h_c	heat transfer conductance or coefficient of coolant, Btu/hr ft^2 ^oF
h_f	heat transfer conductance or coefficient of volume-heat-source fluid, Btu/hr ft^2 ^oF
k	fluid thermal conductivity, Btu/hr ft^2 (^oF/ft)
k_w	pipe wall thermal conductivity, Btu/hr ft^2 (^oF/ft)
L	axial heat exchanger length, ft
m_c	mass flow rate of coolant, lb/hr
m_f	mass flow rate of volume-heat-source fluid, lb/hr
q	heat transfer rate, Btu/hr
q_L	total heat transfer rate for heat exchanger of length L, Btu/hr
r_o	pipe radius or half the distance between parallel plates, ft
t_c	mixed mean coolant temperature of heat exchanger in figure 8, ^oF
t_{ci}	mixed mean coolant temperature at entrance of heat exchanger, ^oF
t_{cL}	mixed mean coolant temperature at exit of heat exchanger, ^oF
$t_{\mathrm{¢}}$	fluid temperature at duct center, ^oF
t_f	mixed mean temperature of the fluid with the volume heat source of the heat exchanger in figure 8, ^oF
t_{fi}	mixed mean temperature of the fluid with the volume heat source at the entrance of the heat exchanger, ^oF

t_{fL} mixed mean temperature of the fluid with the volume heat source at the exit of the heat exchanger, ^oF

t_m mixed mean fluid temperature, ^oF

t_o fluid temperature at duct wall, ^oF

t_1 wall temperature in figure 8, ^oF

t_2 wall temperature in figure 8, ^oF

Δt_{VHS} the wall temperature rise above the mixed mean fluid temperature that exists for the fluid with the volume heat source with no wall heat flux, ^oF

U overall heat transfer conductance or coefficient, Btu/hr ft^2 ^oF

u_m mean fluid velocity, ft/hr

W uniform volume heat source, Btu/hr ft^3

x axial distance, ft

y radial distance from duct wall, ft

γ fluid weight density, lbs/ft^3

δ pipe wall thickness, ft

ν kinematic viscosity, ft^2/hr

Terms

$$M = \frac{W \pi r_o^2}{m_f \ c_{pf}}$$

$$N = \frac{1}{m_f \ c_{pf}} + \frac{1}{m_c \ c_{pc}}$$

$$T = t_f - t_c$$

Dimensionless Moduli

$Nu = \frac{h\ 2r_o}{k}$, Nusselt Modulus for a pipe

$n = \frac{y}{r_o}$

$Pr = \frac{\gamma c_p \nu}{k}$, Prandtl Modulus

$Re = \frac{u_m\ 2r_o}{\nu}$, Reynolds Modulus for a pipe

$\frac{\Delta t_{om}}{\Delta t_{o℄}}$ ratio of the difference between wall and mixed mean fluid temperatures to the difference between wall and centerline temperature for a duct system being cooled at the wall (from reference 3)

INTRODUCTION

Laminar and turbulent forced-convection solutions were derived in references 1 and 2 for the case where fluids with uniform volume heat sources were flowing through circular pipes and between parallel plates respectively, heat was being added to or subtracted from the fluids in a uniform manner at the duct walls These duct systems were postulated to be long so that the thermal and hydrodynamic patterns were established and the physical properties were stipulated to be invariant with temperature The turbulent flow solution for each system was accomplished by separating the general boundary value problem into two simpler ones whose solutions were superposed yielding the solution to the original boundary value problem One boundary value problem defined a flow system with a volume heat source but with no wall heat flux and the second one defined a flow system without a volume heat source but with a uniform wall heat flux In the superposition process, temperatures above datum temperatures are added, for example, the radial temperature distribution above the centerline temperature for the general boundary value problem is obtained by adding the radial temperature distributions above the centerline temperatures for the two specific boundary value problems

The present report gives 1) detailed tabulations of the turbulent temperature profiles[1] for volume-heat-source and wall-heat-flux pipe and parallel plates systemsfor a series of Reynolds and Prandtl moduli and 2) applications of these temperature solutions to two types of convection systems, namely, uniformly and nonuniformly cooled ducts containing flowing fluids with volume heat sources

---

1 Although the detailed radial temperature profiles for turbulent flow had been evaluated at the writing of the earlier reports they were not included at that time, only the dimensionless differences between the wall and mixed mean fluid temperatures were presented because they are generally of more interest

GENERALIZED RADIAL TEMPERATURE PROFILES

The dimensionless radial temperature profiles within fluids having uniform volume heat sources and that are flowing in circular pipes and between parallel plates under turbulent conditions with no wall heat transfer have been evaluated from the solutions given in references 1 and 2 and are tabulated in Tables I and II The corresponding temperature profiles for the case where there are uniform wall heat fluxes but no volume heat sources have been evaluated from Martinelli's solutions (reference 3) and are tabulated in Tables III and IV

Some typical normalized radial temperature profiles for turbulent flow in a pipe for both the volume heat source and wall heat flux cases for Pr = 1 and Pr = 01 are shown plotted in Figures 1, 2, 3, and 4 Note how the shapes of these profiles vary with Reynolds and Prandtl moduli as well as the manner in which heat is added to the fluids The radial temperature distributions are dependent upon the radial heat flow and eddy diffusivity distributions in addition to the boundary layer thicknesses and Prandtl moduli The dimensionless radial heat flow distribution for the wall heat flux case varies linearly from a maximum value at the wall to zero at the duct center, its shape is essentially not a function of Reynolds modulus However, the dimensionless radial heat flow distributions for the volume-heat-source case vary from zero at the wall to a maximum value between the wall and duct center to zero at the duct center, their shapes vary significantly with Reynolds modulus The dimensionless eddy diffusivity profiles vary with radial distance from the wall and Reynolds modulus, and the dimensionless boundary layer thicknesses are dependent on Reynolds modulus The Prandtl modulus significantly influences the thermal resistances in the various flow layers

For example, in Figure 1 (where several temperature profiles are plotted for Pr = 1 for the volume-heat-source case) it can be seen that the fraction of the total temperature drop across the laminar sublayer and buffer layer increases as Reynolds modulus decreases, this occurs because the radial heat flow is proportionately larger in the boundary layers at the lower Reynolds moduli as well as because these layers are thicker under such circumstances Figure 2 reveals several temperature profiles for Pr = 01 for the volume-heat-source case, the thermal resistances are much lower in the boundary layers for low Prandtl moduli fluids and hence the temperature differences across these layers are relatively smaller The temperature profiles in Figure 2 asymptotically approach the laminar flow temperature profile as the Reynolds moduli decrease

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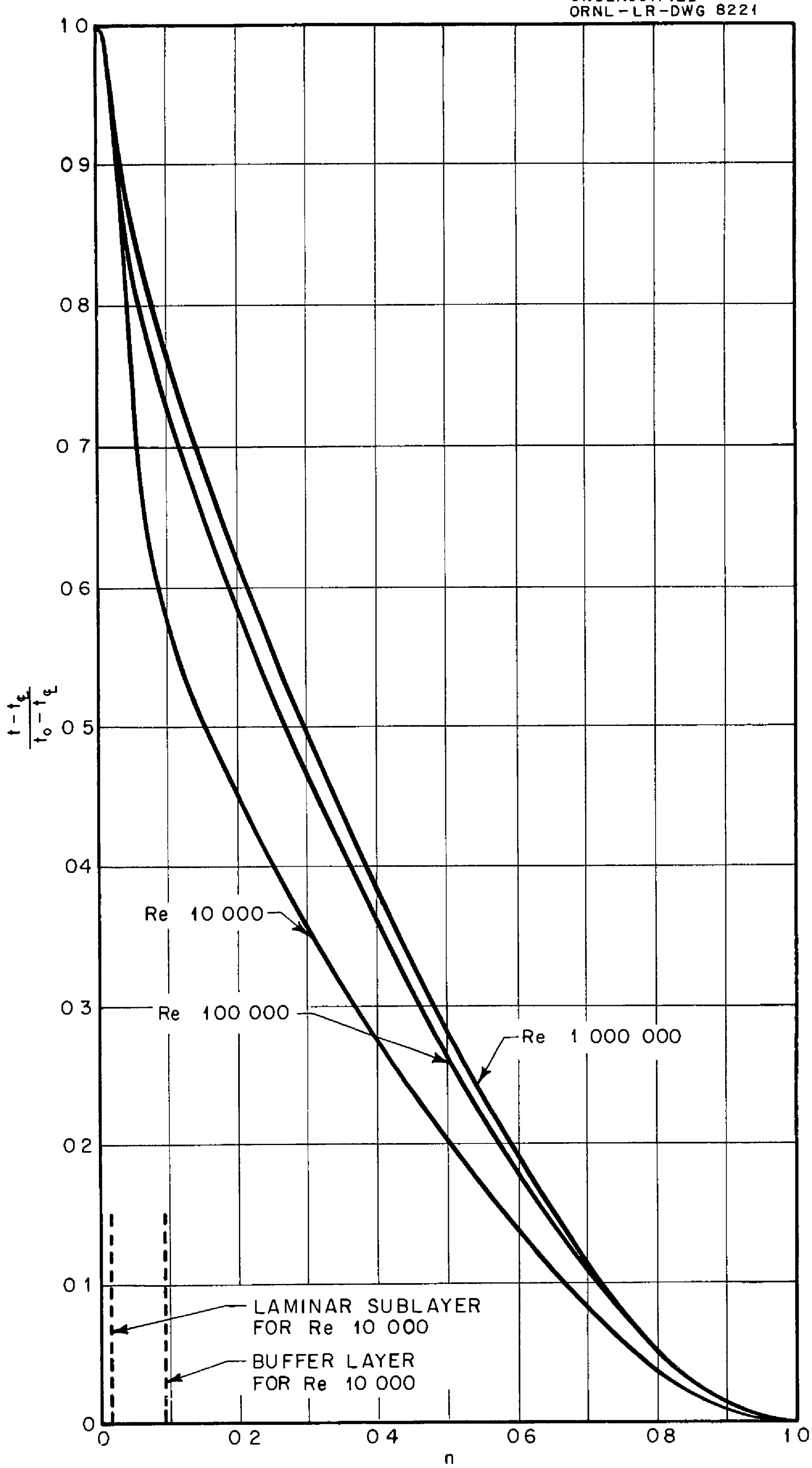

Fig 1 Radial Temperature Distributions Within a Fluid Flowing in a Pipe with a Volume Heat Source in the Fluid and No Wall Heat Flux (Pr 1, Re 10,000, 100,000, 1,000,000)

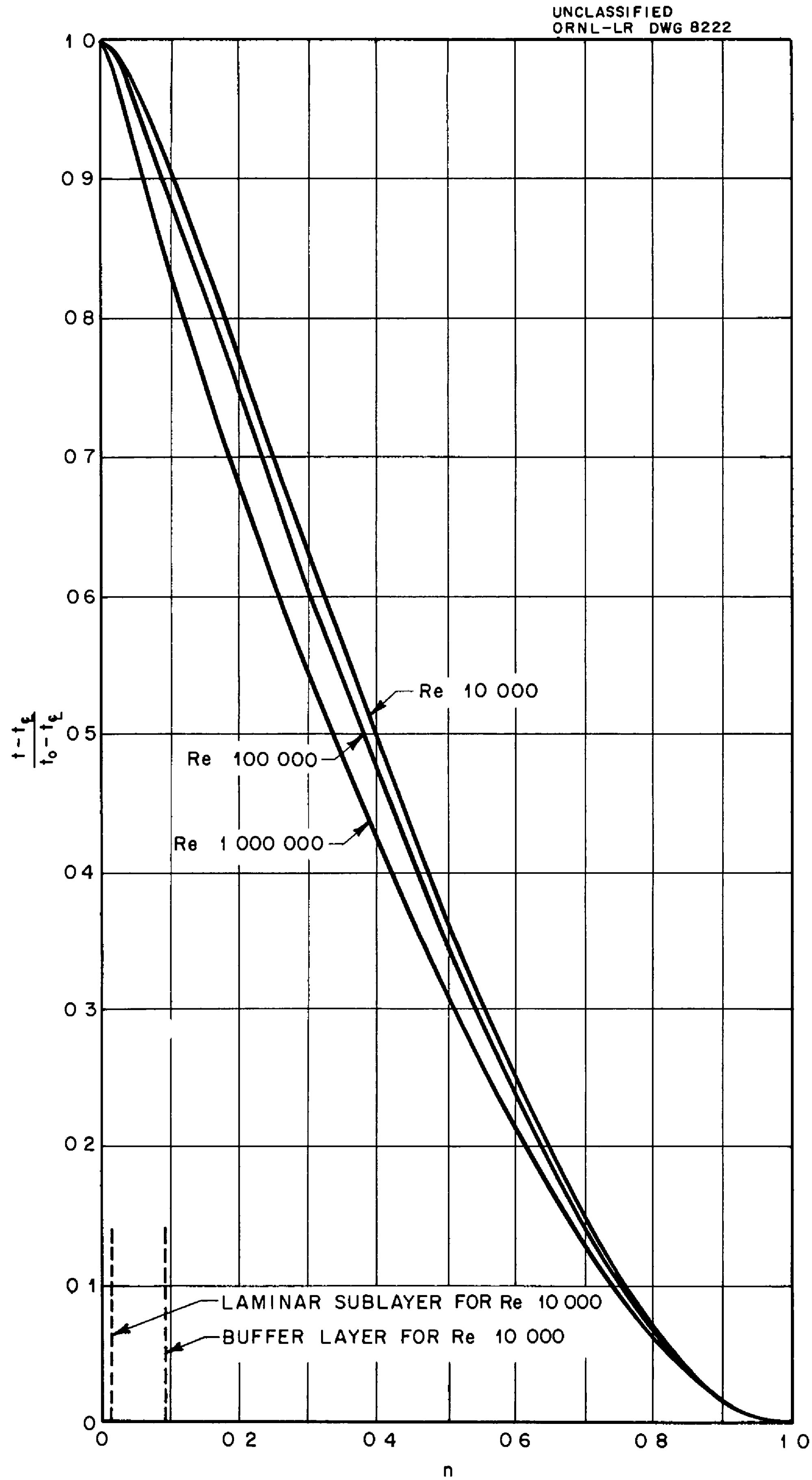

Fig 2 Radial Temperature Distributions Within a Fluid Flowing in a Pipe with a Volume Heat Source and No Wall Heat Flux (Pr 001 Re 10,000, 100,000 1,000,000)

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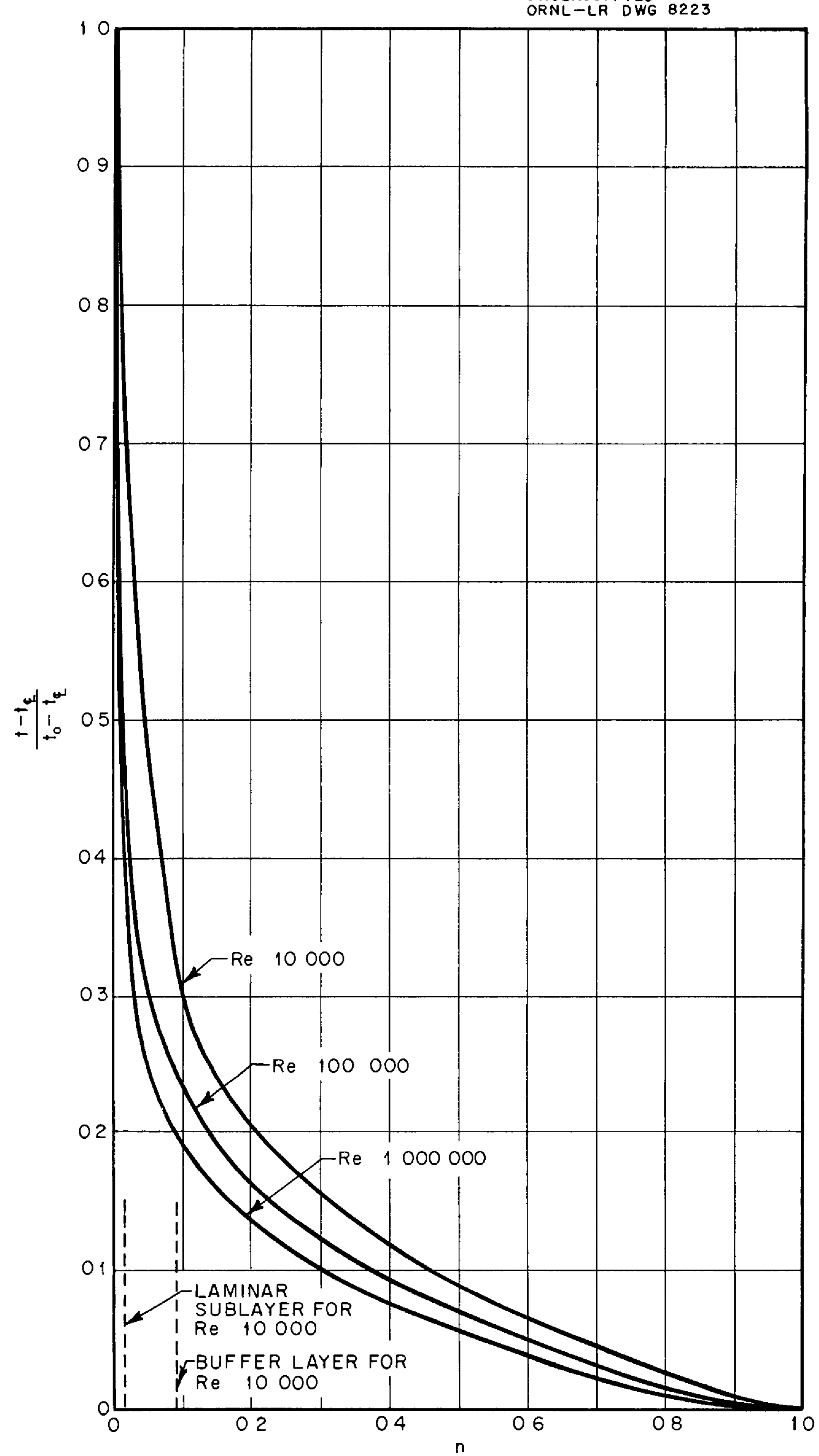

Fig 3 Radial Temperature Distributions Within a Fluid Flowing in a Pipe with Wall Heat Flux but No Volume Heat Source in the Fluid (Pr 1 , Re 10,000 , 100,000 , 1,000 000)

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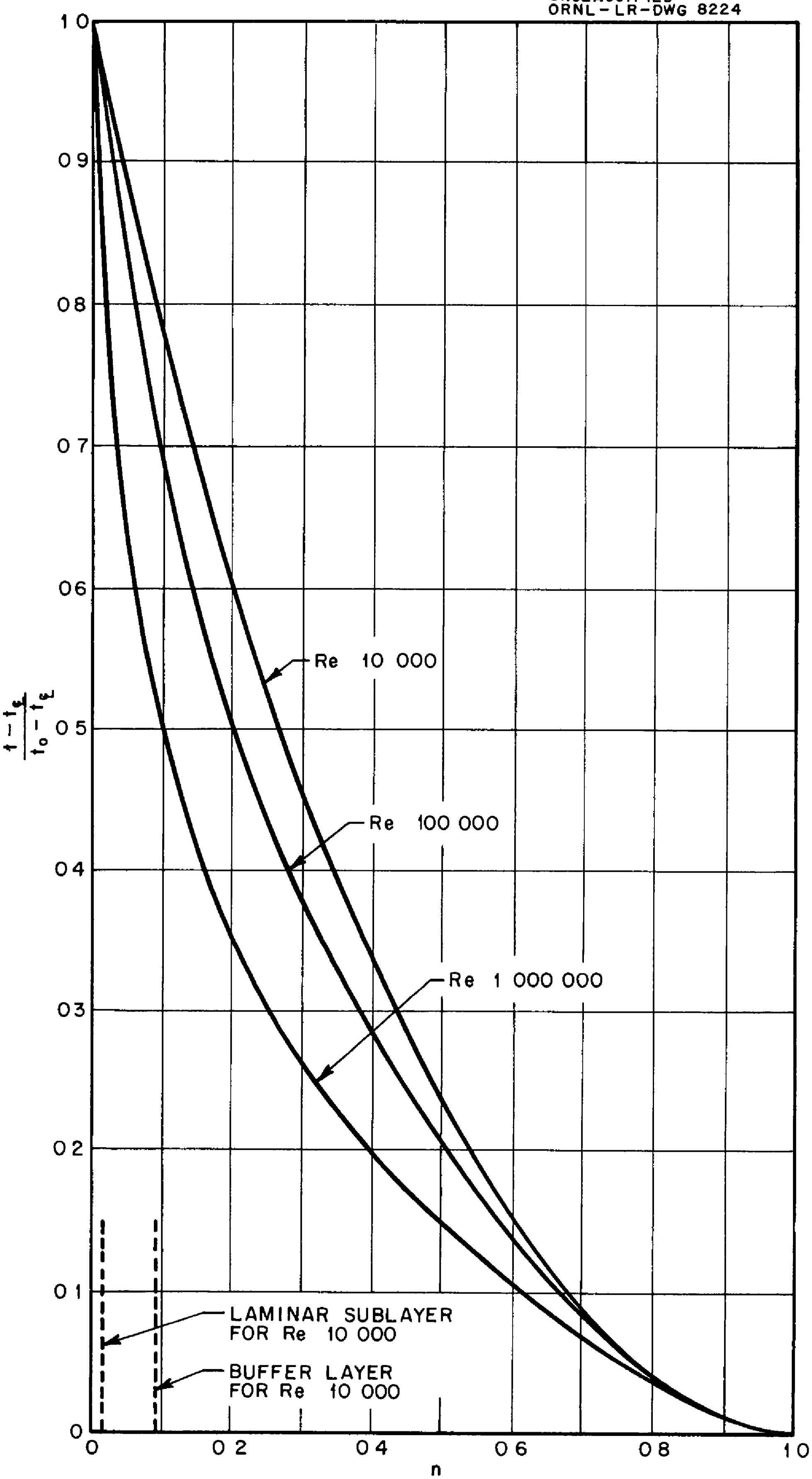

Fig 4 Radial Temperature Distributions Within a Fluid Flowing in a Pipe with Wall Heat Flux but No Volume Heat Source in the Fluid (Pr 001 Re-10,000, 100,000, 1,000,000)

TABLE I

DIMENSIONLESS RADIAL TEMPERATURE DISTRIBUTION FOR A PIPE SYSTEM CONTAINING A UNIFORM VOLUMETRIC HEAT SOURCE BUT HAVING NO HEAT TRANSFERRED AT THE PIPE WALL

$$\frac{t - t_w}{\frac{Wr_o^2}{k}}$$

Re = 5000

n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	4 1703x10^{-2}	3 7591x10^{-2}	5 1021x10^{-3}	1 9956x10^{-3}	1 4197x10^{-3}	1 1438x10^{-3}
025	4 1424	3 7302	4 8143	1 7076	1 1318	8559
05	4 0665	3 6542	4 1949	1 2528	7371	5271
075	3 9601	3 5399	3 7000	1 0407	5978	4179
1	3 8404	3 4200	3 2000	8998	5209	3560
15	3 5652	3 1640	2 5429	6975	3898	2745
2	3 2603	2 8701	2 2000	5759	3240	2290
3	2 6761	2 3479	1 6189	4239	2438	1712
4	2 0772	1 8179	1 2311	3219	1851	1299
5	1 5130	1 3240	8934	2335	1343	0942
6	1 0100	8838	5964	1559	0897	0630
8	2715	2368	1602	0417	0241	0170
1 0	0	0	0	0	0	0

Re = 10,000

n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	3 3566x10^{-2}	2 7680x10^{-2}	2 1094x10^{-3}	7 4364x10^{-4}	5 0511x10^{-4}	4 0573x10^{-4}
025	3 3287	2 7409	1 8643	5 3594	3 1499	2 2433
05	3 2677	2 6800	1 5863	4 2700	2 4659	1 8501
075	3 1797	2 6099	1 3975	3 5204	2 0502	1 4801
1	3 1055	2 5230	1 2192	3 1441	1 8043	1 2679
15	2 9095	2 3401	1 0503	2 7403	1 5401	1 0999
2	2 6927	2 1590	9564	2 4562	1 4082	9888
3	2 0079	1 7560	7505	1 9238	1 1042	7749
4	1 7377	1 3760	5754	1 4739	8461	5936
5	1 2738	1 0081	4208	1 0760	6183	4341
6	8559	6773	2827	7228	4152	2921
8	2319	1835	0768	1956	1131	0795
1 0	0	0	0	0	0	0

TABLE I (Con't)

Re = 100,000						
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	2 3351x10^{-2}	9 6944x10^{-3}	1 7885x10^{-4}	4 9508x10^{-5}	2 9004x10^{-5}	2 1042x10^{-5}
025	2 3202	9 5800	1 5785	4 0097	2 2403	1 5901
05	2 2851	9 2620	1 4655	3 6859	2 1092	1 4750
075	2 2480	8 9508	1 3823	3 4200	1 9804	1 3801
1	2 1831	8 6115	1 2993	3 2660	1 8661	1 3071
15	2 0612	7 9504	1 1613	2 8299	1 6802	1 1800
2	1 9050	7 2058	1 0423	2 6180	1 4963	1 0481
3	1 5790	5 8302	8279	2 0798	1 1892	8331
4	1 2430	4 5379	6405	1 6100	9197	6445
5	9172	3 3436	4713	1 1847	6778	4745
6	6197	2 2588	3182	8010	4591	3209
8	1695	6185	0873	2188	1256	0875
1 0	0	0	0	0	0	0

Re = 1,000,000						
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	1 0818x10^{-2}	1 8160x10^{-3}	2 1151x10^{-5}	5 3623x10^{-6}	3 0820x10^{-6}	2 1733x10^{-6}
025	1 0678	1 7610	1 9582	4 9805	2 8579	1 9703
05	1 0469	1 6760	1 8090	4 5263	2 5839	1 8093
075	1 0179	1 5961	1 6950	4 2400	2 4400	1 7082
1	9808	1 5140	1 6151	4 0405	2 3041	1 6143
15	9118	1 3711	1 4501	3 6201	2 0779	1 4253
2	8270	1 2271	1 2980	3 2474	1 8520	1 2962
3	6720	9799	1 0341	2 5841	1 4729	1 0321
4	5245	7598	8008	2 0012	1 1400	7983
5	3869	5599	5895	1 4730	8395	5872
6	2617	3785	3985	9963	5677	3971
8	0718	1037	1091	2735	1556	1084
1 0	0	0	0	0	0	0

TABLE II

DIMENSIONLESS RADIAL TEMPERATURE DISTRIBUTION FOR A PARALLEL PLATES SYSTEM CONTAINING A UNIFORM VOLUMETRIC HEAT SOURCE BUT HAVING NO HEAT TRANSFERRED AT THE WALLS

$$\frac{t - t_{\text{¢}}}{\frac{W r_o^2}{k}}$$

Re = 5000

n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	6 2533x10^{-2}	5 9943x10^{-2}	1 5040x10^{-2}	6 4605x10^{-3}	4 6881x10^{-3}	3 9211x10^{-3}
025	6 2233	5 9643	1 4741	6 1607	4 3881	3 6211
050	6 1395	5 8804	1 3900	5 3176	3 5451	2 7781
075	6 0125	5 7521	1 2670	4 2975	2 5949	1 8731
1	5 8443	5 5855	1 1500	3 6573	2 1420	1 5230
15	5 4241	5 1671	9436	2 7625	1 6122	1 1281
2	4 9664	4 7223	7767	2 2974	1 2770	8924
3	4 0215	3 8052	5386	1 4769	8795	5850
4	3 1010	2 9120	3886	1 0699	6188	4348
5	2 2518	2 1154	2802	7746	4468	3137
6	1 4983	1 4069	1880	5233	2949	2090
8	4052	3878	0508	1395	0797	0561
1 0	0	0	0	0	0	0

Re = 10,000

n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	4 6965x10^{-2}	4 2910x10^{-2}	6 0926x10^{-3}	2 2948x10^{-3}	1 6414x10^{-3}	1 3060x10^{-3}
025	4 6683	4 2631	5 8087	2 0109	1 3583	1 0219
05	4 5946	4 1889	5 1629	1 5038	9435	6729
075	4 4814	4 0790	4 5597	1 3018	7884	5454
1	4 3513	3 9499	4 0424	1 1288	6733	4828
15	4 0625	3 6658	3 2626	8915	5210	3704
2	3 7516	3 3641	2 6978	7141	4133	2878
3	3 0814	2 7488	2 1032	5558	3209	2220
4	2 4102	2 1412	1 6091	4229	2447	1702
5	1 7654	1 5658	1 1777	3089	1784	1245
6	1 1812	1 0479	7957	2074	1200	0832
8	3170	2802	2187	0567	0323	0225
1 0	0	0	0	0	0	0

TABLE II (Con't)

Re = 100,000						
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	3 1195x10^{-2}	1 7325x10^{-2}	4 4400x10^{-4}	1 2476x10^{-4}	7 5890x10^{-5}	5 7150x10^{-5}
025	3 0995	1 7145	3 8060	9607	5 4550	3 9451
05	3 0615	1 6816	3 5760	8993	5 1097	3 6947
075	3 0075	1 6384	3 3762	8482	4 8251	3 4902
1	2 9426	1 5875	3 1941	8013	4 5648	3 2850
15	2 7776	1 4744	2 8820	7192	4 1003	2 9598
2	2 5917	1 3574	2 5952	6473	3 6898	2 6552
3	2 1637	1 1123	2 0761	5169	2 9552	2 1248
4	1 7117	8728	1 5940	4019	2 3146	1 6551
5	1 2687	6467	1.1881	2974	1 7098	1 2299
6	8585	4387	8081	2000	1 1550	8498
8	2340	1211	2180	0555	3096	2452
1 0	0	0	0	0	0	0

Re = 1,000,000						
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	1 8578x10^{-2}	4 0304x10^{-3}	5 0065x10^{-5}	1 2458x10^{-5}	7 6610x10^{-6}	5 2055x10^{-6}
025	1 8468	3 9216	4 6405	1 1179	6 8949	4 6147
050	1 8179	3 7745	4 3647	1 0498	6 5180	4 3747
075	1 7759	3 6133	4 1364	9959	6 1901	4 1347
1	1 7259	3 4593	3 9206	9438	5 8821	3 9218
15	1 6139	3 1433	3 5386	8519	5 3221	3 5345
2	1 4879	2 8443	3 1861	7578	4 8203	3 1878
3	1 2319	2.2961	2 5563	6119	3 9217	2 5517
4	9724	1 7972	1 9961	4679	3 1104	1 9947
5	7193	1 3341	1 4779	3460	2 3703	1 4800
6	4890	9032	1 0043	2039	1 4349	9974
8	1339	2519	2699	0620	3899	2650
1 0	0	0	0	0	0	0

TABLE III

DIMENSIONLESS RADIAL TEMPERATURE DISTRIBUTION FOR A PIPE SYSTEM HAVING HEAT TRANSFERRED AT THE PIPE WALL BUT CONTAINING NO VOLUMETRIC HEAT SOURCE

$$\frac{t - t_{\not c}}{t_o - t_{\not c}}$$

	Re = 5000					
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9512	9473	7776	5902	5049	4531
05	9134	8957	5928	3353	2412	1899
075	8552	8428	4812	2517	1777	1388
1	8070	7915	4020	2018	1413	1100
15	7108	6883	2905	1382	0959	0743
2	6169	5946	2214	1020	0704	0545
3	4709	4531	1656	0763	0527	0407
4	3462	3336	1261	0581	0401	0310
5	2410	2334	0954	0439	0303	0235
6	1539	1515	0703	0324	0223	0173
8	0124	0396	0307	0141	0098	0076
1 0	0	0	0	0	0	0
	Re = 10,000					
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9499	9418	6425	3768	3744	2175
05	8998	8835	4668	2433	1723	1348
075	8497	8268	3741	1811	1272	0992
1	7998	7717	2918	1404	0981	0763
15	7119	6800	2404	1157	0808	0629
2	6300	5976	2038	0981	0686	0533
3	4816	4539	1526	0734	0513	0399
4	3537	3342	1161	0559	0390	0304
5	2459	2348	0878	0423	0295	0230
6	1577	1535	0647	0311	0218	0169
8	0400	0400	0283	0136	0095	0074
1 0	0	0	0	0	0	0

TABLE III (Con't)

			Re = 100,000			
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9434	8942	3702	1997	1444	1144
05	8896	8106	3006	1622	1173	0929
075	8384	7405	2598	1402	1014	0803
1	7904	6813	2311	1247	0901	0714
15	6980	5802	1904	1027	0743	0588
2	6143	4993	1616	0872	0630	0499
3	4672	3734	1309	0706	0510	0404
4	3438	2777	0920	0496	0359	0284
5	2410	2015	0696	0375	0271	0215
6	1570	1392	0513	0277	0200	0158
8	0418	0462	0224	0121	0087	0069
1 0	0	0	0	0	0	0
			Re = 1,000,000			
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9154	7496	3065	1796	1336	1075
05	8264	6322	2489	1458	1085	0873
075	7586	5546	2152	1261	0938	0755
1	7005	4976	1913	1121	0834	0671
15	5993	4117	1576	0923	0687	0553
2	5171	3498	1337	0783	0583	0469
3	3872	2609	1000	0586	0436	0351
4	2875	1969	0844	0495	0368	0296
5	2050	1470	0576	0337	0251	0202
6	1425	1064	0424	0249	0185	0081
8	0460	0427	0185	0109	0149	0065
1 0	0	0	0	0	0	0

TABLE IV

DIMENSIONLESS RADIAL TEMPERATURE DISTRIBUTION FOR A PARALLEL PLATES SYSTEM HAVING HEAT TRANSFERRED AT THE WALLS BUT CONTAINING NO VOLUMETRIC HEAT SOURCE

$$\frac{t - t_{\mathcal{C}}}{t_o - t_{\mathcal{C}}}$$

n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
			Re = 5000			
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9548	9533	8751	7833	7422	7174
05	9096	9065	7501	5665	4844	4348
075	8667	8601	6317	3831	2819	2245
1	8223	8132	5434	3001	2139	1676
15	7335	7212	4189	2128	1487	1251
2	6338	6300	3305	1604	1111	0860
3	4566	4485	2060	0938	0642	0494
4	3300	3169	1407	0610	0415	0318
5	2110	2050	1064	0462	0314	0241
6	1355	1318	0784	0340	0231	0177
8	0346	0345	0343	0149	0101	0078
1 0	0	0	0	0	0	0
			Re = 10,000			
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9494	9459	7872	6117	5322	4838
05	9001	8917	6027	3450	2489	1962
075	8469	8381	4891	2571	1816	1418
1	8003	7848	4085	2055	1439	1119
15	6940	6794	2948	1404	0974	0754
2	5977	5775	2176	0993	0684	0528
3	4572	4402	1687	0770	0530	0409
4	3358	3237	1284	0586	0403	0312
5	2342	2262	0971	0443	0305	0236
6	1494	1464	0716	0327	0225	0174
8	0378	0381	0313	0143	0098	0076
1 0	0	0	0	0	0	0

TABLE IV (Con't)

			Re = 100,000			
n	Pr = 001	Pr = 01	Pr = 1	Pr = 4	Pr = 7	Pr = 10
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9432	9130	3926	2040	1458	1148
05	8922	8443	3253	1691	1208	0951
075	8456	7827	2813	1462	1045	0823
1	7964	7270	2500	1300	0929	0731
15	7073	6295	2060	1071	0765	0602
2	6244	5464	1748	0908	0649	0511
3	4765	4109	1307	0680	0486	0382
4	3503	3040	0995	0517	0370	0291
5	2446	2175	0753	0391	0280	0220
6	1579	1468	0555	0288	0206	0162
8	0409	0405	0242	0126	0090	0071
1 0	0	0	0	0	0	0
			Re = 1,000,000			
0	1 0	1 0	1 0	1 0	1 0	1 0
025	9262	8131	3270	1864	1373	1099
05	8603	7029	2655	1514	1115	0893
075	8004	6240	2296	1309	0964	0772
1	7456	5625	2041	1164	0857	0686
15	6484	4696	1682	0959	0706	0565
2	5642	3999	1427	0813	0509	0479
3	4251	2980	1067	0608	0448	0359
4	3142	2239	0812	0463	0351	0273
5	2239	1659	0614	0350	0258	0206
6	1501	1185	0453	0258	0190	0152
8	0443	0271	0198	0113	0083	0066
1 0	0	0	0	0	0	0

RADIAL TEMPERATURE PROFILES FOR A PIPE SYSTEM WHOSE WALL IS UNIFORMLY COOLED (AN EXAMPLE)

The temperature profiles tabulated in Tables I, II, III and IV can be used to determine the detailed radial temperature structure in composite convection systems Consider the case where a fluid with a uniform volume heat source is flowing turbulently in a long pipe whose wall is being cooled uniformly along its length The specific conditions of the problem follow

$$W = 0\ 5 \times 10^7 \text{ Btu/hr ft}^3$$

$$r_o = 0\ 15 \text{ ft}$$

$$\left(\frac{dq}{dA}\right)_o = 30{,}000 \text{ Btu/hr ft}^2$$

$$k = 1\ 0 \text{ Btu/hr ft}^2\ {}^oF/\text{ft}$$

$$Re = 10{,}000$$

$$Pr = 1\ 0$$

Determine the detailed radial temperature profile in the fluid

Upon multiplying the dimensionless radial temperature profile given in Table I at Re = 10,000, Pr = 1 0 by the term $\frac{W\ r_o^2}{k} = 1\ 13 \times 10^5\ {}^oF$, a plot of the actual radial temperature profile, above the centerline temperature, can be graphed for the case where a uniform volume heat source exists in the flowing fluid but with no heat transfer occurring at the wall (see Figure 5) Upon multiplying the dimensionless radial temperature profile given in Table III at

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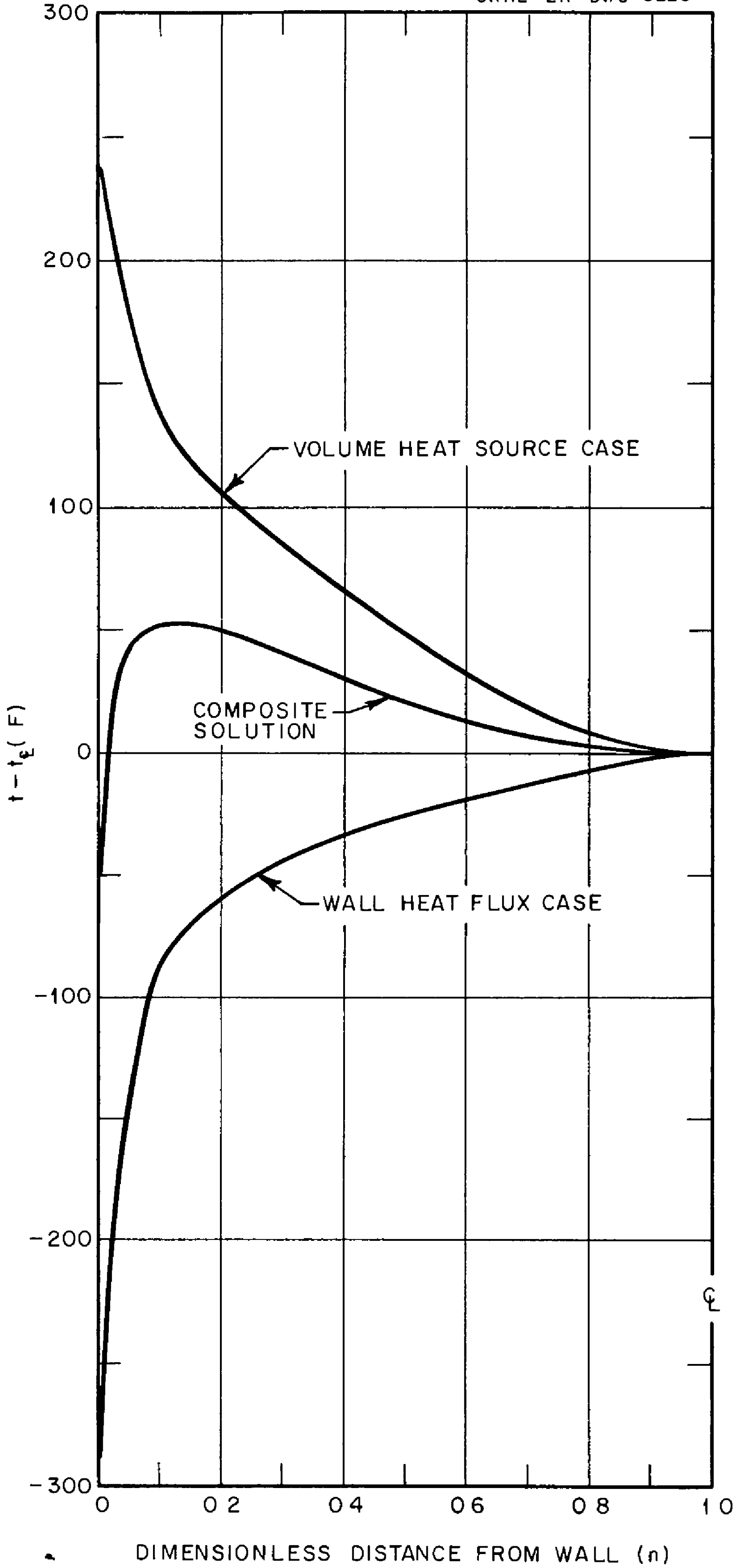

Fig 5 Radial Temperature Distributions for a Pipe System Whose Wall is Uniformly Cooled (An Example)

Re = 10,000, Pr = 1 0 by the negative of the term[2]

$$(t_o - t_{\mathfrak{c}}) = (t_o - t_m) \Bigg/ \left(\frac{\Delta t_{om}}{\Delta t_{o\mathfrak{c}}}\right) = \frac{\left(\frac{dq}{dA}\right)_o}{h} \Bigg/ \left(\frac{\Delta t_{om}}{\Delta t_{o\mathfrak{c}}}\right) = \frac{30,000}{120} \Bigg/ (\ 86) = 290^{o}F,$$

a plot of the actual radial temperature profile, above the centerline temperature, can be graphed for the case where a uniform wall heat flux but no volume heat source exists (see Figure 5) This temperature difference is negative because heat is being extracted from the fluid through the duct wall A superposition of these two curves yields the temperature profile of the composite system above its centerline temperature

2 The functions $\left(\frac{\Delta t_{om}}{\Delta t_{o\mathfrak{c}}}\right)$ from Martinelli's analyses, reference 3, are graphed in Figures 6 and 7 for the pipe and parallel plates system The heat transfer conductances or coefficients can be obtained in references 1, 2, or 3. For the particular problem being considered here, i e , Re = 10,000, Pr = 1 0, k = 1 0, r_o = 15

$$Nu = \frac{h\ 2r_o}{k} = 36$$

$$\text{or} \qquad h = 120 \text{ Btu/hr ft}^2 \ {}^{o}F$$

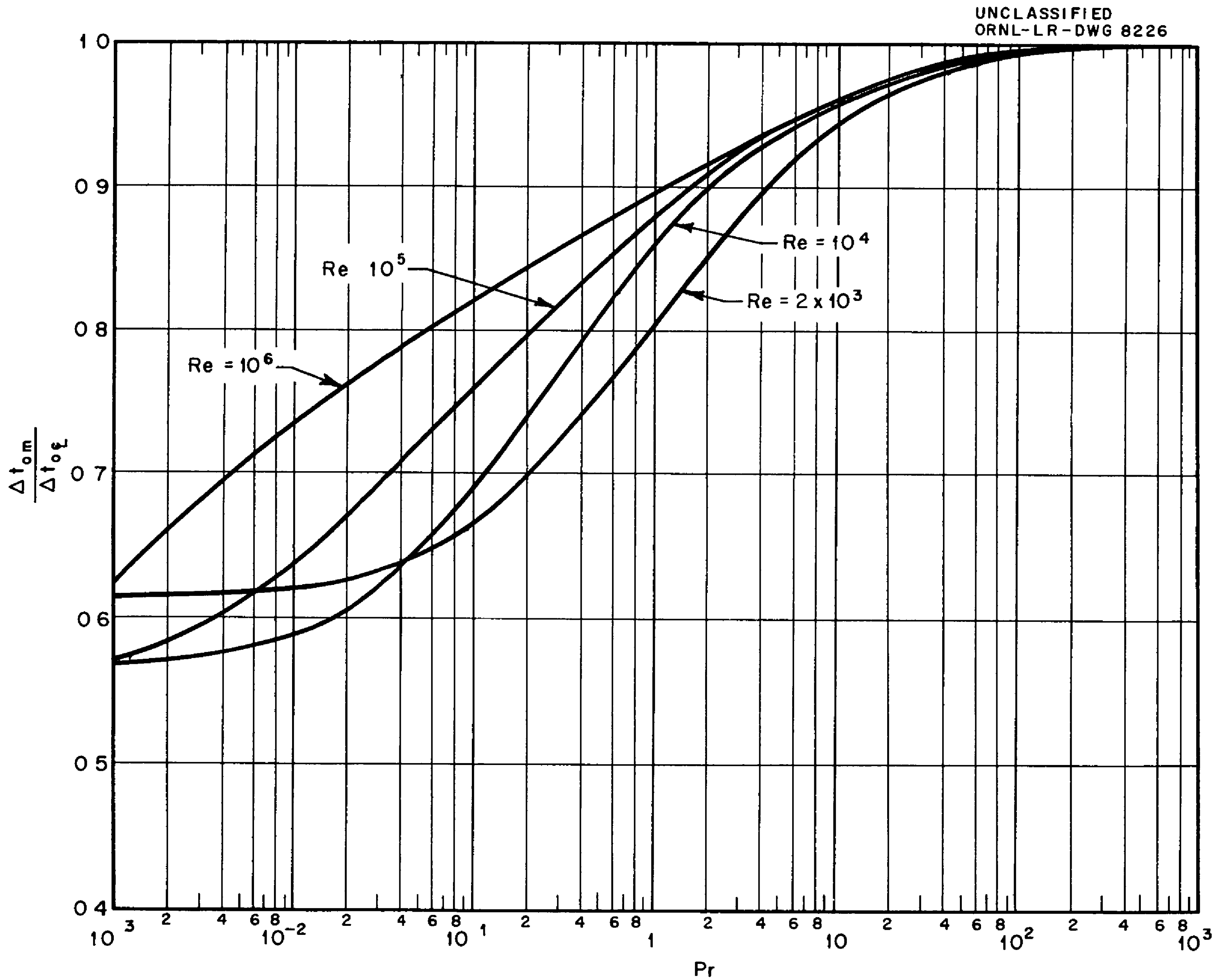

Fig 6 $\frac{\Delta t_{om}}{\Delta t_{o℄}}$ As a Function of Re and Pr for the Wall Heat Flux Case for Pipes (Martinelli)

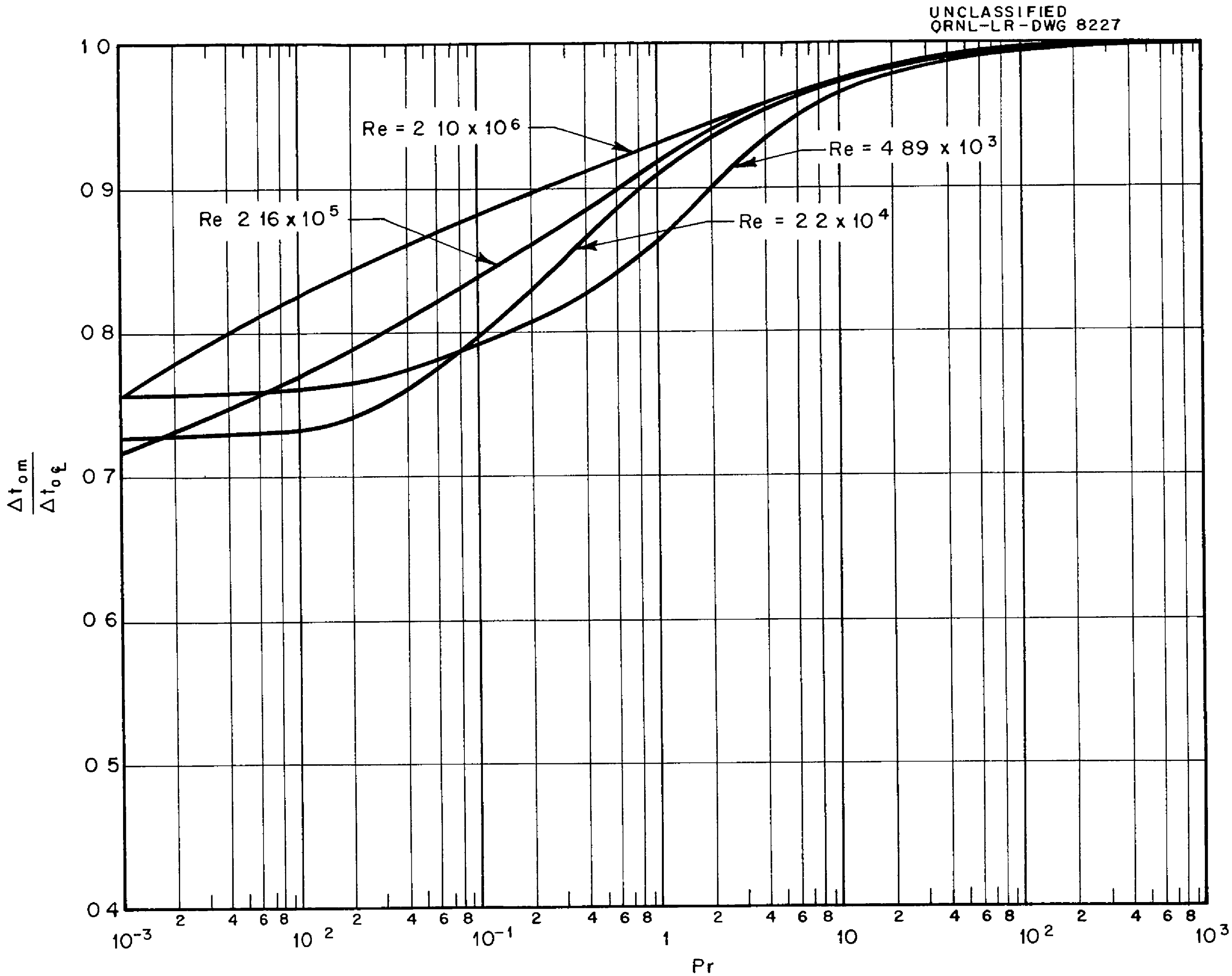

Fig 7 $\frac{\Delta t_{om}}{\Delta t_{o℄}}$ as a Function of Re and Pr for the Wall Heat Flux Case for Parallel Plates (Martinelli)

ANALYSIS OF THE THERMAL STRUCTURE IN A PIPE SYSTEM WHOSE WALL IS NONUNIFORMLY COOLED

Consider the case where a fluid with a uniform volume heat source is flowing turbulently in a pipe whose wall is being cooled, nonuniformly along its length, by a coolant which is flowing in an annular space around the pipe (see Figure 8a) The heat transferred from the wall to the coolant through the differential heat transfer area $2\pi r_o dx$ is (see Figure 8b),

$$dq = h_c \, 2\pi r_o dx \, (t_2 - t_c) \tag{1}$$

The heat transferred through the pipe wall is

$$dq = k_w \, 2\pi r_o dx \, \frac{(t_1 - t_2)}{\delta} \tag{2}$$

The heat transferred from the fluid with the heat source to the wall is[3]

$$dq = h_f \, 2\pi r_o \, dx \left[\Delta t_{VHS} + (t_f - t_1)\right] \tag{3}$$

From equations (1), (2) and (3) one can obtain

$$dq = \frac{2\pi r_o \, dx \left[(t_f - t_c) + \Delta t_{VHS}\right]}{\frac{1}{h_c} + \frac{\delta}{k_w} + \frac{1}{h_f}}$$

$$= U(t_f - t_c + \Delta t_{VHS}) \, 2\pi r_o \, dx \tag{4}$$

Two additional equations arise when making a heat rate balance on the two fluid streams in a length dx (see Figure 8c) The heat gained by the coolant in a parallel flow system is

$$dq = m_c \, c_{pc} \, dt_c \tag{5}$$

---

3 The term Δt_{VHS} represents the wall temperature rise above the mixed mean fluid temperature that exists for the fluid with the volume heat source with no wall heat flux In order to cool the wall temperature to t_1 (see Figure 8b) it is necessary to superpose a wall cooling flux equal to that given in equation (3)

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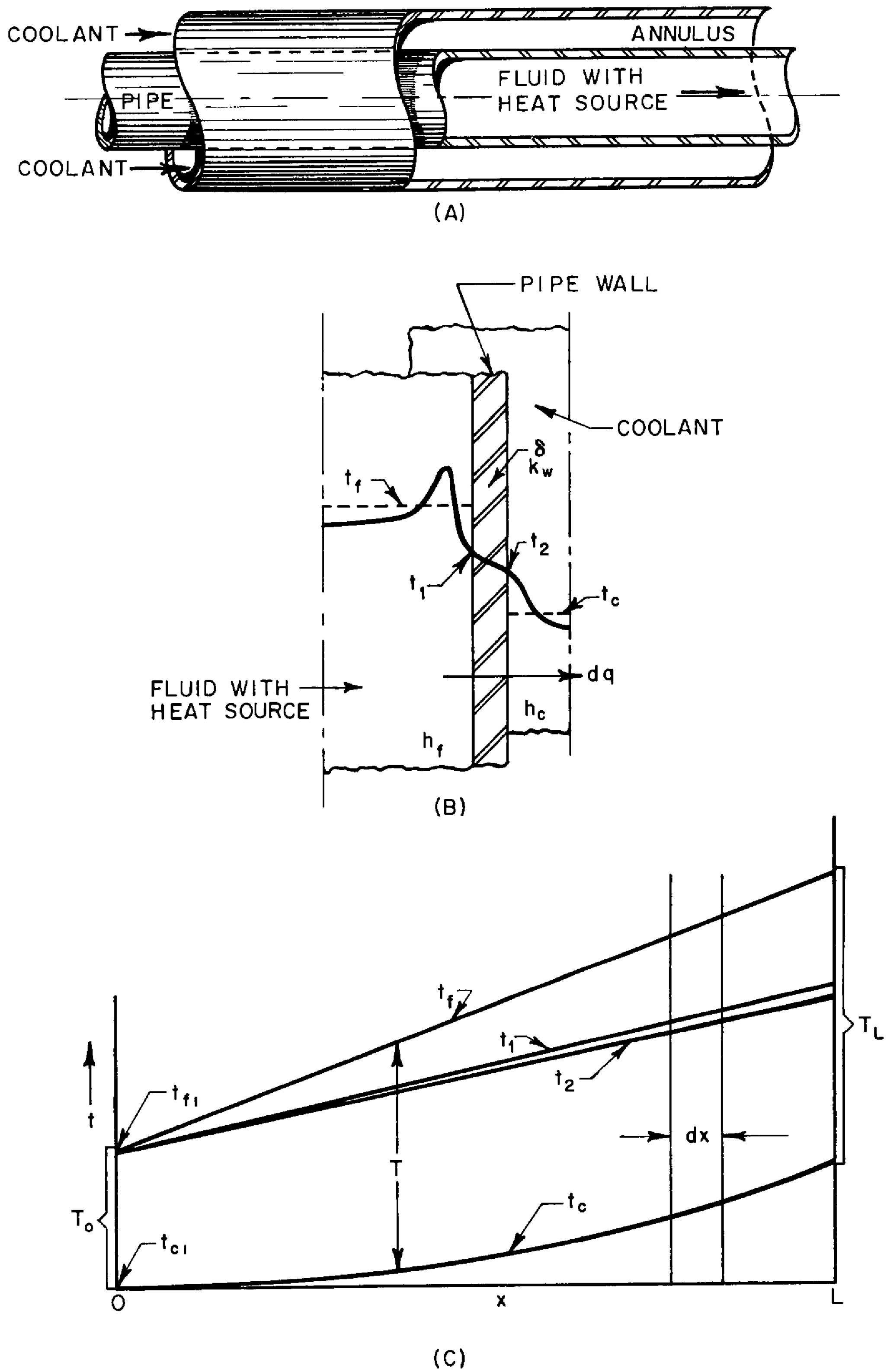

Fig 8 Flow Circuit and Temperature Distributions for a Pipe System Whose Wall is Nonuniformly Cooled

The heat lost by the fluid with the heat source is

$$dq = W\pi r_o^2 \, dx - m_f \, c_{pf} \, dt_f \tag{6}$$

From equations (5) and (6) one can obtain

$$d(t_f - t_c) = -dq \left(\frac{1}{m_f \; c_{pf}} + \frac{1}{m_c \; c_{pc}} \right) + \frac{W\pi r_o^2}{m_f \; c_{pf}} \, dx \tag{7}$$

$$\text{or} \quad dT = -Ndq + Mdx \tag{8}$$

where $T = t_f - t_c$

$$N = \frac{1}{m_f \; c_{pf}} + \frac{1}{m_c \; c_{pc}}$$

$$M = \frac{W\pi r_o^2}{m_f \; c_{pf}}$$

Upon substituting equation (4) into equation (8) there results,

$$dT = -NU(T + \Delta t_{VHS}) \, 2\pi r_o \, dx + Mdx \tag{9}$$

or

$$-\int_o^x dx = \int_{T_o}^{T} \frac{dT}{NU \; 2\pi r_o \; (T + \Delta t_{VHS}) - M}$$

$$- x = \frac{1}{NU \; 2\pi r_o} \ln \frac{NU \; 2\pi r_o T + NU \; 2\pi r_o \; \Delta t_{VHS} - M}{NU \; 2\pi r_o \; T_o + NU \; 2\pi r_o \; \Delta t_{VHS} - M} \tag{10}$$

$$\text{or} \quad T + \Delta t_{VHS} = \left(T_o + \Delta t_{VHS} - \frac{M}{NU \; 2\pi r_o} \right) e^{-NU \; 2\pi r_o x} + \frac{M}{NU \; 2\pi r_o} \tag{11}$$

The heat transfer rate q can be obtained by substituting equation (11) into equation (4) and integrating

$$\int_o^q dq = 2\pi r_o U \int_o^x \left[\left(T_o + \Delta t_{VHS} - \frac{M}{NU \; 2\pi r_o} \right) e^{-NU \; 2\pi r_o x} + \frac{M}{NU \; 2\pi r_o} \right] dx$$

$$\text{or} \quad q = \frac{1}{N} \left(T_o + \; t_{VHS} - \frac{M}{NU \; 2\pi r_o} \right) \left(1 - e^{-NU \; 2\pi r_o x} \right) + \frac{M}{N} \, x \tag{12}$$

The coolant temperature variation can be obtained by substituting equation (12) into equation (5),

$$\int_{t_{ci}}^{t_c} dt_c = \frac{1}{m_c\ c_{pc}} \int_o^q dq$$

or $$t_c - t_{ci} = \frac{1}{m_c\ c_{pc}\ N}\left(T_o + \Delta t_{VHS} - \frac{M}{NU\ 2\pi r_o}\right)\left(1 - e^{-NU\ 2\pi r_o x}\right) + \frac{M}{N\ m_c\ c_{pc}}\ x \qquad (13)$$

The mixed mean fluid temperature variation of the fluid containing the heat source can be obtained by substituting equation (12) into equation (6),

$$\int_{t_{fi}}^{t_f} dt_f = -\frac{1}{m_f\ c_{pf}} \int_o^q dq + \frac{W\pi r_o^{\ 2}}{m_f\ c_{pf}} \int_o^x dx$$

or $$t_f - t_{fi} = -\frac{1}{m_f\ c_{pf}\ N}\left(T_o + \Delta t_{VHS} - \frac{M}{NU\ 2\pi r_o}\right)\left(1 - e^{-NU\ 2\pi r_o x}\right) - \frac{M}{Nm_f\ c_{pf}}\ x$$

$$+ \frac{W\ \pi r_o^{\ 2}}{m_f\ c_{pf}}\ x \qquad (14)$$

The surface temperatures of the heat exchanger wall may be obtained from equations (1), (2), and (4),

$$t_2 - t_c = \frac{U(t_f - t_c + \Delta t_{VHS})}{h_c} \qquad (15)$$

and

$$t_1 - t_2 = \frac{U(t_f - t_c + \Delta t_{VHS})}{\frac{k_w}{\delta}} \qquad (16)$$

The terms t_c and $(t_f - t_c + \Delta t_{VHS})$ were previously derived in equations (13) and (11), respectively

TEMPERATURE STRUCTURE IN A PIPE SYSTEM WHOSE WALL IS NONUNIFORMLY COOLED (AN EXAMPLE)

An illustrative example of a pipe-annulus system whose wall is nonuniformly cooled by parallel coolant flow follows

Given,

$W = 0\ 5 \times 10^7$ Btu/hr ft^3

$r_o = 0\ 15$ ft

$k_f = 1$ Btu/hr ft ^oF

$c_{pf} = 1\ 0$ Btu/lb ^oF

$Pr_f = 1$

$m_f = 2{,}360$ lb/hr

$Re_f = 10{,}000$

$t_{ci} = 0$

$t_{fi} = 150^oF$

$T_o = 150^oF$

$\delta = 0\ 005$ ft

$k_w = 20$ Btu/hr ft ^oF

$L = 4$ ft

$m_c = 1600$ lb/hr

$c_{pc} = \ 5$ Btu/lb ^oF

$h_c = 123$ Btu/hr ft^2 ^oF

Determine the total amount of heat transferred to the coolant flowing through the annulus as well as the temperature structure of the system

$$\Delta t_{VHS} = 1\ 3 \times 10^{-3} \frac{Wr_o^2}{k} = 1\ 3 \times 10^{-3} \frac{(\ 5 \times 10^7)(2\ 25 \times 10^{-2})}{1} = 146^oF$$

$Nu_f = 36$

$h_f = 120$ Btu/hr ft^2 ^oF

$$N = \frac{1}{m_f\ c_{pf}} + \frac{1}{m_c\ c_{pc}} = \frac{1}{(2360)(1)} + \frac{1}{1600\ (5)} = 0\ 00167 \frac{hr\ ^oF}{Btu}$$

$$M = \frac{W\pi r_o^2}{m_f\ c_{pf}} = \frac{(\ 5 \times 10^7)(\pi)(2\ 25 \times 10^{-2})}{(2360)\ (1)} = 150 \frac{^oF}{ft}$$

$$\frac{1}{U} = \frac{1}{h_c} + \frac{\delta}{k_w} + \frac{1}{h_f} = \frac{1}{123} + \frac{005}{20} + \frac{1}{120} = 0\ 0228\ \frac{hr\ ft^2\ {}^oF}{Btu}$$

$U = 59\ 7$ Btu/hr ft^2 ^{o}F

Thus, from equation (12),

$$q_L = \frac{1}{0\ 00167}\left[150 + 146 - \frac{150}{(0\ 00167)(59\ 7)2\pi(\ 15)}\right]\left[1 - \frac{1}{e^{(0\ 00167)(59\ 7)2\pi(\ 15)(4)}}\right]$$

$$+ \frac{(150)(4)}{(0\ 00167)}$$

$$= 115{,}500 \text{ Btu/hr}$$

Also, from equation (13)

$$t_{cL} - t_{ci} = \frac{115{,}000}{1600\ (\ 5)} = 145^oF$$

and from equation (14)

$$t_{fL} - t_{fi} = -\frac{115{,}000}{(2360)(1)} + \frac{(0\ 5 \times 10^7)\pi(2\ 25 \times 10^{-2})(4)}{(2360)\ (1)} = 551^oF$$

The detailed temperature structure of the pipe-annulus system is graphed in Figure 9 The fraction of the total heat generated within the fluid flowing in the pipe which is extracted by the coolant flowing in the annulus is

$$\frac{q_{coolant}}{q_{generated}} = \frac{q_L}{W\pi r_o{}^2 L} = \frac{115{,}000}{(\ 5 \times 10^7)\pi(2\ 25 \times 10^{-2})4} = 0\ 082$$

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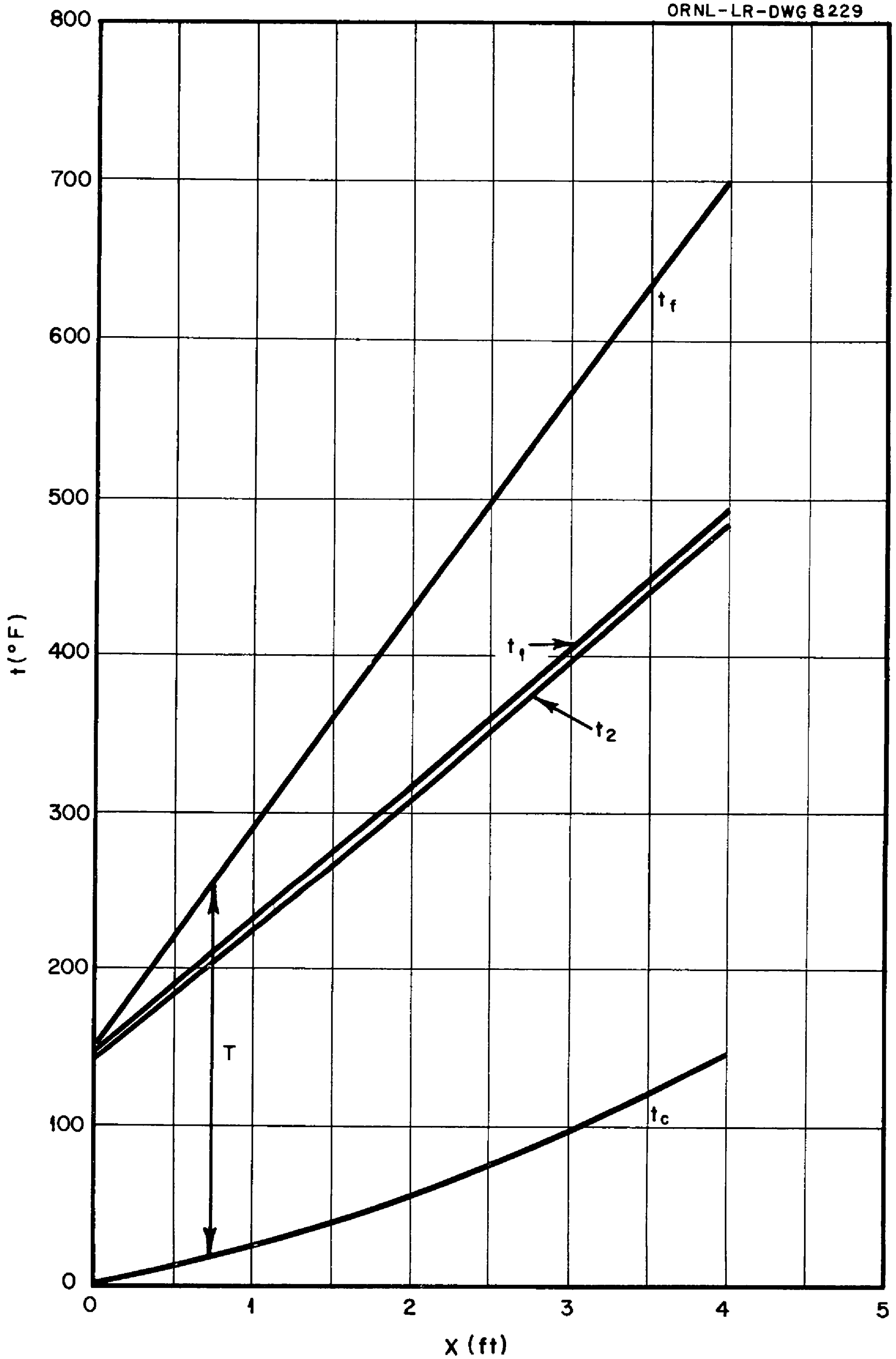

Fig 9 Temperature Structure in a Pipe System Whose Wall is Nonuniformity Cooled (an example)

CLOSING REMARKS

The forced-flow volumetric-heat-source solutions which were previously developed were applied to two specific heat exchange systems They may also be applied to other types of convection systems, several of which are suggested below

1) Parallel plates system whose wall is nonuniformly cooled The analysis presented for the nonuniformly cooled pipe system may be modified to obtain a solution for a parallel plates system by replacing the pipe heat transfer area, $2\pi r_0 dx$, by a corresponding one for the parallel plates system

2) Pipe and parallel plates systems whose walls are being cooled by fluids having volumetric heat sources The analysis presented for the nonuniformly cooled pipe system may be modified to obtain the temperature solutions for general convection systems in which the coolants also contain volumetric heat sources Under these circumstances a Δt_{VHS} term for the coolant is included in equation (1), and a volumetric heat source is included in equation (5), the analysis is accomplished as before The new equation for T now contains a modified form of the parameter M and also a Δt_{VHS} for the coolant has been added The same modifications occur in the equation for the heat transfer rate, q

3) Pipe and parallel plates systems which are being nonuniformly cooled by counter flow In this case it is merely necessary to insert a minus sign in equation (5) and carry it through the remaining analysis

This report has stressed only the turbulent flow regime although both laminar and turbulent flow analyses were presented in references 1 and 2 Applications for laminar-flow volume-heat-source systems parallel those presented here for turbulent flow It is interesting to note, however, that the heat extraction or cooling rates necessary to reduce wall temperatures to mixed mean fluid or centerline temperatures in the case of laminar flow are much greater than those for turbulent flow For example, it is necessary to extract 33 1/3 percent of the heat generated within a laminarly flowing fluid in a pipe system to bring its wall temperature down to the centerline temperature, whereas for turbulently flowing ordinary fluids the corresponding heat extraction rate is only several percent

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