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SUBJECT: Delayed Neutron Losses in Circulating Fuel Reactors – MSCR Memo No. 6

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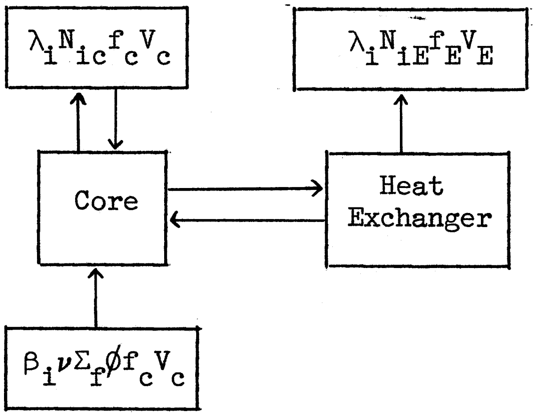
Abstract

Equations which describe delayed neutron losses in external loops of circulating fuel reactors were derived. A working equation and the necessary input data for calculating delayed neutron losses by an equilibrium reactor code such as ERC-5 are given.

DELAYED NEUTRON LOSSES IN CIRCULATING FUEL REACTORS

Circulating fuel reactors lose neutrons because some of the delayed neutrons are emitted outside of the core. These losses depend on core residence time, external loop residence time, and decay characteristics of the precursors.

A symbolic representation of the system is



where

λ_{ij} = decay constant of the i^{th} precursor from fissionable material j,

β_{ij} = number of i^{th} precursors formed per fission neutron from fissionable material j,

N_{ijc} = atoms of i^{th} precursor per unit volume of fuel stream in the core resulting from fissions in material j,

N_{ijE} = atoms of i^{th} precursor per unit volume of fuel stream in the external loops resulting from fissions in material j,

f_c = volume fraction of fuel in the core,

f_E = volume fraction of fuel in the external loops,

V_c = core volume,

V_E = external loop volume,

$\nu_j \Sigma_{fj} \phi f_c V_c$ = rate of production of fission neutrons in the core from fissionable material j.

The precursor concentrations are described by these equations:

$$\frac{dN_{ijc}}{dt_c} = \beta_{ij} \nu_j \Sigma_{fj} \phi - \lambda N_{ijc} , \tag{1}$$

$$\frac{dN_{ijE}}{dt_E} = - \lambda N_{ijE} , \tag{2}$$

where

t_c = time in the core,

t_E = time in the external loops.

The boundary conditions are:

$$N_{ijc}(T_c) = N_{ijE}(0) , \tag{3}$$

$$N_{ijc}(0) = N_{ijE}(T_E) , \tag{4}$$

where

T_c = time for the fuel stream to pass through the core,

T_E = time for the fuel stream to pass through the external loops.

The solutions to Eqs. (1) and (2) are:

$$N_{ijc} = \frac{\beta_{ij} \nu_j \Sigma_{fj} \phi}{\lambda_{ij}} (1 - e^{-\lambda_{ij} t_c}) + N_{ijc}(0)\, e^{-\lambda_{ij} t_c} , \tag{5}$$

$$N_{ijE} = N_{ijE}(0)\, e^{-\lambda_{ij} t_E} . \tag{6}$$

Note that the precursor production rate is assumed constant for the fuel stream during its stay in the core. This idealized case would exist only for uniform power density along the fuel stream or for core residence times which are short compared to the half-life of the precursor.

The boundary conditions become:

$$\frac{\beta_{ij}\nu_j\Sigma_{fj}\phi}{\lambda_{ij}}\left(1 - e^{-\lambda_{ij}T_c}\right) + N_{ijc}(0)\, e^{-\lambda_{ij}T_c} = N_{ijE}(0)\ , \tag{7}$$

$$N_{ijc}(0) = N_{ijE}(0)\, e^{-\lambda_{ij}T_E}\ . \tag{8}$$

Eliminating $N_{ijc}(0)$ in Eqs. (7) and (8) and substituting the result in Eq. (6) gives:

$$N_{ijE} = \frac{\beta_{ij}\nu_j\Sigma_{fj}\phi}{\lambda_{ij}}\,\frac{\left(1 - e^{-\lambda_{ij}T_c}\right)e^{-\lambda_{ij}t_E}}{\left[1 - e^{-\lambda_{ij}(T_c + T_E)}\right]}\ . \tag{9}$$

The rate of decay of precursors in the external loops is:

$$f_E V_E \frac{\Delta N_{ijE}}{T_E} = \frac{\beta_{ij}\nu_j\Sigma_{fj}\phi f_E V_E\left(1 - e^{-\lambda_{ij}T_c}\right)\left(1 - e^{-\lambda_{ij}T_E}\right)}{\lambda_{ij}T_E\left[1 - e^{-\lambda_{ij}(T_c + T_E)}\right]}\ . \tag{10}$$

The total rate of precursor decay (at equilibrium) is $\beta_{ij}\nu_j\Sigma_{fj}\phi f_c V_c$. Thus the fraction of the delayed neutrons which appear in external loops is:

$$\frac{\dfrac{\Delta N_{ijE} f_E V_E}{T_E}}{\beta_{ij}\nu_j\Sigma_{fj}\phi f_c V_c} = \frac{f_E V_E\left(1 - e^{-\lambda_{ij}T_c}\right)\left(1 - 2^{-\lambda_{ij}T_E}\right)}{f_c V_c \lambda_{ij} T_E\left[1 - e^{-\lambda_{ij}(T_c + T_E)}\right]}\ . \tag{11}$$

Since $\frac{f_E V_E}{f_c V_c} = \frac{T_E}{T_c}$, Eq. (11) becomes:

$$\frac{\beta_{ijE}}{\beta_{ij}} = \frac{(1 - e^{-\lambda_{ij}T_c})(1 - e^{-\lambda_{ij}T_E})}{\lambda_{ij}T_c\left[1 - e^{-\lambda_{ij}(T_c + T_E)}\right]} \quad . \tag{12}$$

For using these results in an equilibrium reactor code such as ERC-5,[1] the term $\nu_j\Sigma_{fj}\phi$ may be replaced by a neutron production rate given by

$$N_j C_j^f \nu_j \ ,$$

where

C_j^f = reaction rate coefficient for fissions in material j.

Using this in Eq. (12) gives the following result for the number of neutrons lost in the external loops per neutron produced:

$$\text{losses} = \sum_j N_j C_j^f \nu_j \sum_i \frac{\beta_{ij}(1 - e^{-\lambda_{ij}T_c})(1 - e^{-\lambda_{ij}T_E})}{\lambda_{ij}T_c\left[1 - e^{-\lambda_{ij}(T_c + T_E)}\right]} \quad . \tag{13}$$

The necessary constants for Th^{232}, U^{233}, U^{235}, U^{238}, and Pu^{239} are:[2]

Group	λ_i (sec^{-1})	Th^{232}	U^{233}	U^{235}	U^{238}	Pu^{239}
1	0.0128	0.00085	0.00020	0.0003	0.00015	0.0001
2	0.0315	0.0035	0.00075	0.0018	0.0017	0.0006
3	0.125	0.0045	0.00105	0.0022	0.0028	0.00045
4	0.325	0.0120	0.00075	0.0023	0.0071	0.00085
5	1.55	0.0045	0.00025	0.0007	0.0042	0.0003
6	4.5	0.0009	--	0.0002	0.0015	--
		0.02625	0.0030	0.0075	0.01745	0.0023

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[1] L. G. Alexander, *ERC-5 Program for Computing the Equilibrium States of Two-Region, Thorium Breeder Reactors*, ORNL-CF-60-10-87 (Oct. 20, 1960).

[2] A. M. Weinberg and E. P. Wigner, *The Physical Theory of Neutron Chain Reactors*, p. 136, The University of Chicago Press, Chicago, 1958.

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